MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XCI. DECEMBER 12, 1930. No. 2

Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Mark Herbert Joseph Bennett, Lieutenant, R.N.R., Hoglands Wood, Dunmow, Essex ;

John Hasselkus, Director of "Ross Limited," Optical Works, 3 North Side, Clapham, London, S.W. 4 ;

Hector Archibald McDonald, B.Sc., Instructor Lieutenant-Commander, R.N., Royal Naval College, Greenwich, London, S.E. 10 ; and

Alexander Raven Thomson, 30 Overstrand Mansions, London, S.W. 11.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Geoffrey Hamer Bairstow, Ardengair, 121 Home Park Road, Wimbledon, London, S.W. 19 (proposed by F. J. M. Stratton) ;

I. James Bews, 151 Earl Street, Scotstoun, Glasgow (proposed by W. H. Coombs) ;

Roger Daniel Hewart Jones, B.A., Eryl Daf, Radyr, near Cardiff (proposed by F. J. M. Stratton) ; and

Julie Marie Vinter-Hansen, M.Sc., University Observatory, Östervald 3, Copenhagen (proposed by L. J. Comrie).

Sixty-eight presents were announced as having been received since the last meeting, including :—

Astrographic Catalogue: Catania, R. Observatory, vol. 5, pts. 3, 4; vol. 6, pts. 3, 4; Rome, Specola Vaticana, Appendix 3, pt. 2.

Eddington (Sir A. S.), The Rotation of the Galaxy (Halley Lecture, 1930).

Lemaître (l'Abbé G.) (1) Un univers homogène de masse constante et de rayon croissant. (2) Le mouvement varié d'un solide, d'après la théorie de la relativité. (3) La grandeur de l'espace.

Pulkovo Observatory, Refraction Tables, 3rd edition.

Mr. W. M. Witchell and Dr. A. J. Thompson were appointed auditors of the Treasurer's accounts for 1930.

Apsidal Motion in Y Cygni and other Stars.

By H. N. Russell and R. S. Dugan.

1. Variations in period are all too common among eclipsing binaries ; but the cases in which they can be explained by motion of the periastron are rare. For Y Cygni, the spectroscopic observations of Plaskett and Redman * are in full agreement with the photometric results of Dugan and Waterfield,† and no doubt exists as to the fact of apsidal motion. Later observations indicate a slightly slower motion—revolution of the line of apsides taking 5700 periods of the close pair (or 46·8 years). The cause of this motion has been variously attributed to the ellipticity of the eclipsing stars and to the attraction of a third body in the system. Both may obviously co-operate to produce it.

2. *Ellipticity of the Stars.*—The components of Y Cygni are practically equal in mass. The eclipses are equally deep, and the loss of light at each is approximately 44 per cent. of that of the whole system. Hence the components are of equal surface brightness, and their radii cannot differ by more than 11 per cent. For the present purpose we may assume the radii to be equal, leaving the question of any difference to the detailed discussion which one of us has in progress.

For equal stars, which keep the same faces toward one another, the equation for the motion of periastron ‡ becomes

$$P/P' = 16kr^5/a^5 \quad . \quad . \quad . \quad . \quad (1)$$

where P is the orbital period, P' that of the apse, a the radius of the orbit, r that of a star, and k depends on the internal distribution of density.

* *M.N.*, **90**, 754, 1930. † *H.B.*, **806**, 1928 ; *Pop. Ast.*, **36**, 563, 1928.
‡ *M.N.*, **88**, 642, 1928.

The oblateness due to rotation is given by

$$e = (1 + 2k)r^3/a^3 \quad . \quad . \quad . \quad . \quad (2)$$

while the ellipticity of a star's equator, arising from tidal attraction, is $\frac{3}{2}e$.

In the present case we have $P'/P = 5700$, $r/a = 0{\cdot}20$ (from a preliminary solution of the light curve), whence $k = 0{\cdot}034$* and $\frac{3}{2}e = 0{\cdot}0128$. This gives the variation due to ellipticity, if the star discs are of uniform brightness. For "complete" darkening toward the limb, the effect is 1·6 times as great, or 0·020. The "reflection" effect, however, diminishes even these small values. According to Milne† the light reflected by component 2 at phase angle ϵ is $L_1 r_2^2 f(\epsilon)/a^2$ where $f(\epsilon)$ is 0·708, 0·226, and 0·00 when $\epsilon = 0, \pi/2$, and π. Summing for the two stars, we find that, in the present case, the light is less at quadrature ($\epsilon = \pi/2$) than at conjunction by 0·0051 $(L_1 + L_2)$. Hence the observable range in brightness due to ellipticity should be 0·008 for uniform discs and 0·015 for complete darkening. The former is likely to be nearer the truth for so hot a star. From about 560 observations made by one of us outside eclipse, the corresponding constant is found to be $+ 0{\cdot}002 \pm 0{\cdot}008$. The observations are therefore consistent with the theoretical ellipticity necessary to account for the whole motion of the apsides; but they do not suffice to evaluate it with certainty.

3. It is worth noting in this connection that for β Aurigæ, which also has equal components, with $r/a = 0{\cdot}146$, Stebbins‡ finds an ellipticity-variation of $0{\cdot}010 \pm 0{\cdot}003$ of the whole light. If k had the value found above for Y Cygni, the theoretical variation, with allowance for reflection, would be 0·002 for uniform and 0·005 for darkened discs. To match the observed variation a much larger k and smaller central condensation are required. For completely darkened discs Stebbins's observations demand $k = 0{\cdot}37$, intermediate between the values for Jupiter and the Earth, while for uniform discs $k = 0{\cdot}90$—an incredible value demanding an increase of density toward the surface. Without stressing these numerical results, the data are clearly unfavourable to a high central condensation. Shapley's conclusion§ that the ellipticities of eclipsing stars in general (for uniform discs) are in close agreement with Darwin's theoretical values for masses of homogeneous and incompressible fluid points in the same direction.

4. *Effect of a Third Body.*—The existence of a third body in the system of Y Cygni is made probable by a change in the radial velocity of the centre of gravity of the close pair, amounting to 10 km./sec. between 1919–21 and 1928–29. Mr. Redman suggests that the attraction of this body may account for the motion of the periastron.

This problem has been discussed theoretically by Slavenas.‖ When

* This is much smaller than the value found in the previous paper (where Shapley's $r/a = 0{\cdot}167$ was adopted). It corresponds to a central density 16 times the mean, instead of 6, as there deduced.

† *M.N.*, **87**, 54, 1926.

‡ *Ap. J.*, **34**, 112, 1911.

§ *Ibid.*, **38**, 173, 1913.

‖ *Yale Transactions*, **6**, 41, equation (I), 1927.

the orbit of the distant companion is circular, the masses are M_0, M_1, M_2, and the mean motions of the close and wide pairs are n, n', he finds

$$\frac{\omega}{n} = \frac{3}{4}km^2 + \frac{225}{32}k^2m^3 + \left(\frac{849}{32} + \frac{675}{128}k\right)k^2m^4 . \quad . \quad (3)$$

where ω is the mean motion of periastron, $m = n'/n$, and

$$k = M_2/(M_0 + M_1 + M_2).$$

As Redman notes, the mass of the companion must be relatively small. The eclipses show that each of the main components gives at least 0·44 of the light of the whole system, leaving at most 0·12 for the third body, and the form of the light curve is best represented if the light of the third body is assumed to be insensible.

From the mass-luminosity relation, it follows that the third body is, at most, about half as massive as either of the others, so that $k \lesssim \frac{1}{5}$.

Assuming various values of k we may now calculate the values of m corresponding to the assumption that the apsidal motion arises entirely from this perturbation. The period of the third body is then

$$P'' = 2{\cdot}996/m \quad . \quad . \quad . \quad . \quad (4)$$

The relative velocity of the close pair (reduced to a circular orbit) is 482 km./sec. The orbital velocity of the centre of mass of the close pair will then be

$$V = 482k\{m/(1-k)\}^{\frac{1}{3}} \quad . \quad . \quad . \quad (5)$$

There is no evidence of changes in the inclination of the close pair, hence the orbits are probably nearly coplanar, and γ for the close pair will vary with semi-amplitude V.

The times of minima will be affected by a light-equation due to this motion, with a semi-amplitude (in minutes of time)

$$A = VP''/1306 \quad . \quad . \quad . \quad . \quad (6)$$

We then find, with $\omega/n = 1/5700$,

k.	M_2.	m.	P''.	V.	A.
			d.	km./sec.	m.
$\frac{1}{5}$	8·8 ⊙	0·0321	93·3	33·0	2·35
$\frac{1}{7}$	5·8	0·0394	76·0	24·6	1·43
$\frac{1}{11}$	3·5	0·0495	60·5	16·2	0·75
$\frac{1}{21}$	1·75	0·0688	43·5	9·6	0·32
$\frac{1}{51}$	0·70	0·1078	27·8	4·55	0·10

It appears that a third body, to account for the apsidal motion, must have a period of three months or less. Its effects on the time of eclipse are too small to be detected, but those on the mean radial velocity of the close pair may be observable if the third body is not too small. Plaskett's observations of 1919–21, the only ones so far published, are too scattered to afford a decisive test. They appear,

however, to indicate a progressive change in velocity of some 25 km./sec. between 1920 June 20 and August 9.

It may be noted that a companion (of given mass) too remote to account for the whole of the apsidal motion will give a longer period, a smaller velocity range for the centre of mass, and a larger light-equation. If the period is years in length, as suggested by Redman, the perturbations of the eclipsing system will be negligible, but there may be a considerable light equation, with a semi-amplitude as great as half an hour, which could readily be detected, if the earlier observations were sufficiently numerous and precise.

It is to be hoped that this very interesting—and very elusive—star will be followed up until the true cause—or causes—of the apsidal motion have been settled. Intensive spectroscopic observation during one summer should go far toward a decision.

5. The most striking result of this discussion is that a third body, even of considerable mass, must have a short period in order to produce as great an apsidal motion as a very small ellipticity of the principal stars. It should be remembered that Y Cygni is one of the stars in which apsidal motion should be slow, since r/a is smaller than usual. For the majority of eclipsing variables the apsidal periods should be but a few years or months—unless the central concentration is greater than is indicated in the cases discussed above. The slow alterations in period through decades, which so often are found, can therefore hardly be accounted for in this way, and remain an outstanding problem in celestial mechanics.

Princeton University Observatory :
1930 *November.*

The Spectrum of H.D. 20336 in Camelopardalis (Sp. Type B3neη) throughout a complete Cycle (1925–1930). By William J. S. Lockyer, M.A., Ph.D., Major (late R.A.F.). (Plates 6, 7.)

In a previous communication to the Society * in 1926 I gave a brief summary of the early observations of bright lines recorded in the spectrum of H.D. 20336 in Camelopardalis (1900 R.A. $3^h\ 11^m{\cdot}2$. Dec. $+\ 65°\ 17'$. Mag. 4·76. Spectral type B3neη).

Special attention was drawn to the observations of R. H. Curtiss, made in 1912 and 1916 at Ann Arbor, who alone had stated a periodicity for this star, deducing that " at least two bodies are involved forming a system with a long orbital period—possibly about 2·3 years," basing this conclusion on radial velocity measures.

Some observations made at the Norman Lockyer Observatory at Sidmouth during two months in 1923, three months in 1924, and two months in 1925 led me to conclude that these later observations would not satisfy Curtiss's period, but, assuming a period of about the same

* *M.N.R.A.S.*, **86**, 607, 1926.

order of magnitude as that put forward by Curtiss, they conformed more to one of 1·4 years, satisfying fairly well Curtiss's observations.

It was then considered advisable to institute a new series of observations, as continuous as possible, in order to determine an accurate value for the variation, and the star has been kept under close observation ever since.

The new observations demonstrate clearly that both the above periods are entirely incorrect and that the new value obtained is of the order of about 4·3 years or about 1568 days.

While this investigation was in progress, Curtiss * announced that he had deduced a new period of 1690 days (4·63 years). At a later date (1929) he stated † that "the latest Ann Arbor period is 1710 days." This star, he further states, "is one of the more regular of these variables. The curve for this star is practically a sine curve."

In the first reference just given he stated that he had deduced the period from the curve of variation of the ratios of the intensities of the two bright components of the hydrogen lines, nearly the same method of procedure as employed by me, but he does not seem to have published any curves or details of his observations. Only the actual value of his latest period has been published, in order, as he states, to present "for the benefit of other observers the more recent results of several of these stars which are being followed at Ann Arbor." This suggests that no detailed account of the research had then (1929) been published.

In reply to a communication from me in June of last year Curtiss stated that he had much material on Be-type stars in manuscript, but now his untimely death may possibly prevent their publication for some little time.

While several years must elapse before an accurate value of the period for this star can be determined from Sidmouth observations, there seems no object in delaying the publication of the results so far obtained, since they clearly approximately corroborate the longer period advocated by Curtiss.

As the work has involved a very considerable expenditure of time at the telescope, at least 400 hours, in securing this series of photographs, such a perfectly independent investigation of the periodic changes in the spectrum may be a useful contribution to the history of this star.

All the photographs have been secured with the same instrument as before, namely, the 9-inch prismatic camera, having a focal length of 120 inches and a prism of 45° angle, the dispersion being 46·0 mm. between H_β and H_δ.

The plates used were the Ilford Iso-Zenith (H. and D. 700), and the exposures usually given were about two hours.

It may be stated that the star is circumpolar, so that it can be photographed all the year round, but during the months April to June its position near lower culmination during the night hours renders it rather difficult at times to secure very good photographs.

* *Pop. Ast.*, **33**, No. 8, 537, 1925. † *Ibid.*, **37**, No. 10, 578, 1929.

The programme of work originally laid down was to try to secure a good photograph every week, but as time went on it was found that the spectrum did not change so rapidly as was expected, since it appeared that a much longer period was involved. For this reason the interval was extended to a fortnight, and this fortnight was later changed to one month. In fact it was eventually found that one good photograph taken every month would be quite sufficient to determine the periodicity with fair accuracy.

The total number of photographs of the spectrum secured for this discussion amounts to 209, extending from 1923 September 10 to 1930 November 25, but as stated above no such great number was really needed.

Two reproductions of the spectrum accompanied my previous paper * on this star, but a brief general description of it may be given here.

It is of the spectral type B3neη. It consists of broad absorption lines of hydrogen, very strong and nebulous helium lines, a weak and nebulous ionised magnesium line at λ 4481, a moderate but sharp [K] line, and some faint but sharp enhanced bright metallic lines similar in origin to those that appear strong in ϕ Persei, the spectral type of which is Boneη. The broad absorption lines of hydrogen, as far as H_η, have superimposed on them two bright sharp lines, placed symmetrically about the centre with a sharp absorption line between them. These bright components change their intensity in relation to each other; sometimes they are equally bright, sometimes the violet component is stronger than the red and *vice versa.*

The method of determining the periodicity of the star was to make an eye-estimate with a magnifying glass ($\times 2$) of the relative intensity of the two bright components in each of the hydrogen lines $H\beta$ and H_γ for all the negatives secured. Thus it was estimated whether the red component was equal to (R = V), a little brighter than (R > V(L)), brighter than (R > V), much brighter than (R > > V), and, lastly, very much brighter than (R > > > V) the violet component. The same procedure was adopted with regard to the violet component in relation to the red, namely, V = R, V > R(L), V > R, V > > R, and V > > > R. It was found that by this method quite good estimates could be made from relatively weak negatives.

As a check on the results the relative intensities of the bright hydrogen components for the line H_β were determined by means of the wedge micrometer. In this case only the very best negatives were employed, as the definition of the lines had to be good.

Thus two quite independent investigations were made. The *eye-estimates* were made by the writer, and Mr. D. L. Edwards made the *wedge measures*, selecting two of the best negatives in each month when available, each measure being the mean of eight settings.

The fact that the relative intensity of the bright components of H_γ behaved very similarly to those of H_β led one to omit the values of the former from the following table.

* *M.N.R.A.S.*, **86**, 609, Plate 14, 1926.

For the two methods of measurement each observer divided the negatives into three classes as regards his idea of the quality of the definition of the bright components for his own work, and named them good (*g*), moderate (*m*), and poor (*p*). Thus in columns (3) and (6) in Table I., the number preceding any of the letters *g*, *m*, and *p* means the number of negatives examined for the resulting value of the "estimate" or "measure." Thus for 1923 December, the "estimate" was made from two "good" and one "moderate" negative, and the "measure" made from one "poor" negative.

In the following table, column (1) gives the year, (2) the month, (3) the numbers and qualities of the negatives, (4) the eye-estimate of the intensities of the bright components of H_β as previously described, (5) the wedge measures of the intensities of the same components, the plus sign meaning that the red was stronger than the violet component, and, lastly, column (6) the numbers and qualities of the negatives used for the wedge measures.

TABLE I.

Relative Intensities of the Bright Components of Hβ.

(Quality of negatives: *g* = good; *m* = moderate; *p* = poor.)

Year.	Month.	Eye-estimates.		Wedge Measures.	
		No. and Quality.	Estimate.	Measure.	No. and Quality.
(1).	(2).	(3).	(4).	(5).	(6).
1923	Sept.	3*g*	R>>V	+1·8	2*m*
	Dec.	2*g* 1*m*	R>V	+0·5	1*p*
1924	Jan.	1*m*	R=V ?		
	Oct.	1*p*	V>R ?		
	Nov.	1*p*	V>>R ?		
1925	Oct.	1*g*	V>>>R	−3·5	1*g*
	Dec.	1*m*	V>>>R		
1926	July	1*m*	V=R ?		
	Aug.	2*g* 2*m*	V>R (L)	−0·2	1*g* 1*m*
	Sept.	4*g* 1*m*	R=V	+0·1	1*m* 1*p*
	Oct.	3*g* 3*m*	R>V (L)	+0·5	2*g*
	Nov.	3*g* 1*m*	R>V	+0·3	1*m*
	Dec.	8*g* 1*m*	R>V	+0·7	1*m* 1*p*
1927	Jan.	7*g*	R>>V	+0·9	1*g*—*m*
	Feb.	3*g*	R>>V	+1·8	1*m* 1*p*
	March	4*g* 5*m*	R>>>V	+1·5	1*g* 1*m*
	April	2*m*	R>>>V	+2·4	1*m*—*p*
	May	2*m* 2*p*	R>>>V		
	June	2*g* 3*m*	R>>>V		
	July	1*g* 2*m*	R>>>V		
	Aug.	2*g* 3*m*	R>>>V	+2·3	1*m* 1*p*
	Sept.	4*g* 2*m*	R>>>V	+1·7	1*g*—*m*

TABLE I.—*continued.*

Year.	Month.	Eye-estimates.		Wedge Measures.	
		No. and Quality.	Estimate.	Measure.	No. and Quality.
(1).	(2).	(3).	(4).	(5).	(6).
1927	Oct.	5*g* 3*m*	R>>>V	+1·8	1*m*
	Nov.	2*g* 2*m*	R>>>V		
	Dec.	1*g*	R>>V	+1·2	1*p*
1928	Jan.	6*g* 4*m*	R>>V	+0·9	1*m*—*p*
	Feb.	2*g* 1*m*	R>>V	+1·3	1*g*—*m*
	March	1*g* 2*m*	R>>V	+0·5	1*m*
	April	1*m*	R>>V		
	Aug.	2*p*	R=V ?		
	Sept.	2*m* 2*p*	V=R	0·0	2*p*
	Oct.	2*m*	V=R ?		
	Nov.	1*g* 1*m*	V=R	0·0	1*m*
	Dec.	2*g*	V>R	−0·3	2*g*—*m*
1929	Jan.	1*g*	V>>R		
	Feb.	2*g*	V>>R	−0·8	1*p*
	March	3*g*	V>>R	−1·2	1*p*
	April	2*g*	V>>>R	−2·1	1*m*
	May	1*m*	V>>>R	−2·7	1*m*
	June	1*p*	V>>>R ?		
	July	2*p*	V>>>R		
	Aug.	2*g* 1*p*	V>>>R	−2·9	1*m*—*p*
	Sept.	2*g*	V>>>R		
	Oct.	1*g*	V>>>R	−2·9	1*p*
	Dec.	1*g*	V>>>R		
1930	Jan.	1*g*	V>>>R	−3·2	1*m*
	Feb.	1*g*	V>>>R		
	March	1*g*	V>>>R		
	April	1*g* 1*m*	V>>>R	−2·7	1*g*—*m*
	May	1*m* 1*p*	V>>R	−2·4 ?	1*p*
	June	1*g*	V>>R	−1·6	1*m*
	July	1*m*	V>>R		
	Aug.	1*m* 3*p*	V>>R	−1·1 ?	1*p*
	Sept.	3*g*	V>>R	−1·4	2*g*—*m*
	Oct.	1*g*	V>R	−0·9	1*g*
	Nov.	1*g* 1*p*	V>R (L)	−0·5	1*m*

A graphical representation of the variations of the values of the "estimates" and "measures" is shown in fig. 1. The former variation is displayed in the upper curve (broken line), the scale for which is

indicated on the left-hand side with broken lines carried through the curve. The latter variation is given below as a continuous line and its scale is represented on the right with continuous lines carried through the curve. The time scale for both curves is the same and is shown below, J representing January in each year. The mean value for each month is recorded against the respective month on the time scale.

The figures against each dot represent the number of negatives from which the values were derived as shown in Table I., and the black dots indicate the most trustworthy values.

It will be seen from the " estimate " curve in fig. 1 that two horizontal series of points represent the maximum and minimum of the curve, corresponding to the estimates R > > > V and V > > > R respectively. It was found difficult to differentiate between these points in each series by the eye-estimation method. Much greater differentiation is shown at the maximum and minimum by the wedge method of measurement.

The smooth curves passing through the points were drawn for the two sets quite independently of each other, each curve being made to satisfy as nearly as possible the individual observations. The only restrictions made were that the least satisfactorily determined points of maximum and minimum in 1923 and 1925 respectively should be made to read the same values on the intensity scales as the best determined maximum and minimum for each curve separately. Thus, in the case of the eye-estimate curve the best determined maximum occurring in 1927 on about July 1 reaches a point a little above the R > > > V line: the point of the maximum in 1923 is made to reach the same level, and this occurs on about 1923 March 1. Also the minimum of the same curve, which occurs on 1925 July 1 is made to reach a little below the V > > > R line, the same as the better determined minimum in 1929 October 15. A similar procedure was adopted as regards the curve of the wedge measures.

The chief features which the curves indicate are :

(1) The epochs of maxima and minima are practically the same in both curves.

(2) A decided change of curvature or " hump " is indicated on the downward side of the curves, very pronounced in the wedge measures curve, but only slightly suggested in the eye-estimate curve.

(3) The interval from minimum to maximum is less than that from maximum to minimum, the deficit being about 107 days.

An important feature, restricted to the curve of the wedge measures, is that the maximum point is at a much smaller distance above the zero line than the minimum is below the zero line, the actual scale readings being 2·4 and 3·4 respectively.

In order to state more precisely the dates of the various epochs as determined from the eye-estimate curve, their Julian day numbers are given as follows :—

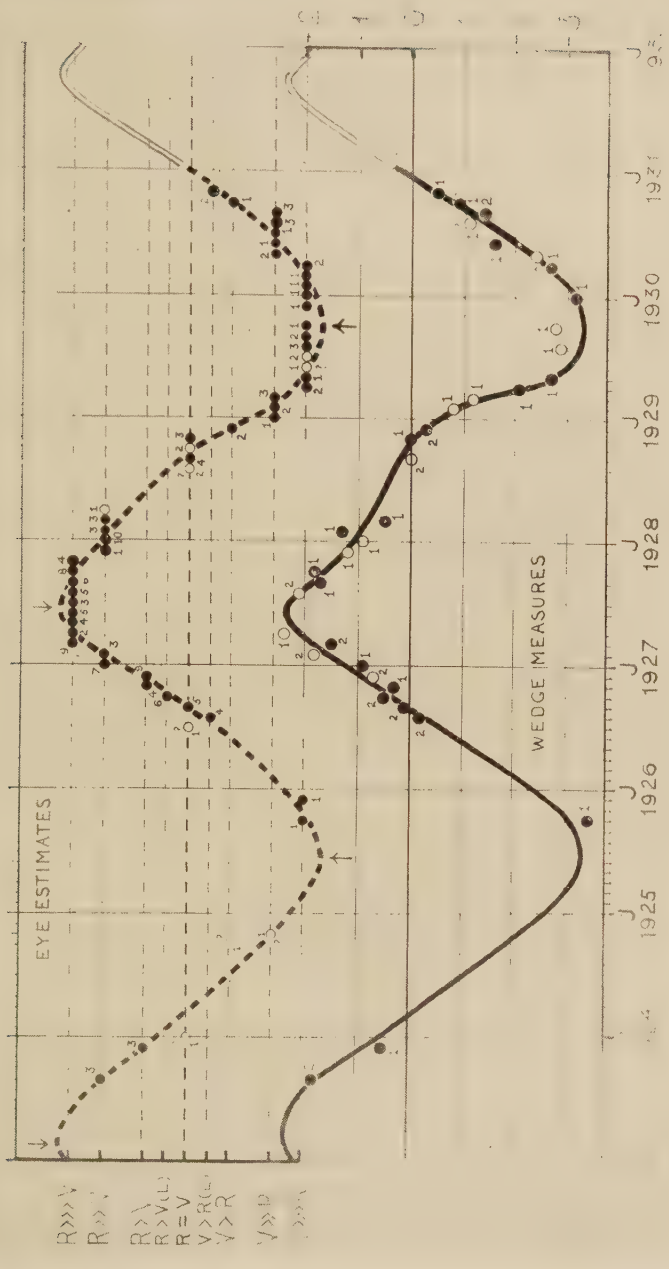

EYE ESTIMATES
WEDGE MEASURES
R>>>V
R>>V
R>V
R>V(L)
R=V
V>R(L)
V>R
V>>R
V>>>R
J 1925
J 1926
J 1927
J 1928
J 1929
J 1930
J 1931

Fig. 1.

R > > > V	Maximum. . .	1923 Mar. 1 J.D. 2423480
V > > > R	Minimum . . .	1925 July 1 4333
V = R	Upward on Curve .	1926 Sept. 15 4804
R > > > V	Maximum. . .	1927 July 1 5063
V > > > R	Minimum . . .	1929 Oct. 15 5900
V = R	Upward on Curve .	1931 Jan. 15 6357

The value of the length of the period can be determined separately from three intervals, namely, those between the two maxima, the two minima, and, lastly, between the upward portions of the curve where they intersect the zero (V = R) line. The last-mentioned interval should give perhaps the most accurate value.

The following are the durations in days so determined. They are put in tabular form, in which are incorporated Curtiss's two periods :—

Length of Period.

	Days.		Days.
Lockyer—			
From Max. to Max.	1583		
,, Min. to Min.	1567	mean	1567·7
,, Zero (up) to Zero (up)	1553		
Curtiss—			
1st Period stated	1690		
2nd ,, ,,	1710		1710·0
		Difference	142·3

It will be seen that the Sidmouth value of the period falls short of Curtiss's deduction by about 142 days. This difference is sufficiently great to prevent his curve from conforming satisfactorily to the Sidmouth observations.

It was mentioned near the commencement of this communication that Curtiss stated that the curve he obtained for this star was "practically a sine curve." Now this does not seem to be the case, according to the Sidmouth observations, in two respects. First, the intervals between minimum to maximum and maximum to minimum are not equal : in the former case the interval has a duration of 730 days and in the latter 837 days, thus displaying a difference of 107 days.

Secondly, there is a decided change of curvature or "hump" on the downward side of the curve, which of course is not inherent in a sine curve.

In order to compare our results with such a sine curve, with a period of 1710 days conforming to Curtiss's deduction and having the same amplitude as that of the eye-estimate curve, a drawing of such a curve was made on tracing paper and placed over the eye-estimate curve.

As the actual dates of the epochs of maxima or minima of Curtiss's sine curve have not been published, this curve was first placed over the eye-estimate curve so as to make the best fit with the points on the upward side of the curve, commencing in 1925. The sine curve was found not only to be considerably late at the following maximum and

minimum epochs, but most of the points on the downward side were late as well.

Again, placing the sine curve so that its maximum coincided with the epoch of maximum of the eye-estimate curve in 1927, then the sine curve was very much too early on *all* the points on the upward side of the eye-estimate curve, but conformed fairly well with those on the downward side. At the minimum in 1929 both curves were in good agreement.

The facts that the Sidmouth curves were drawn independently from data treated in two quite separate ways, that they both agree in their main features, but both equally differ from Curtiss's sine curve, suggest that both the shape of his curve and the length of its period must be modified to make them conform to the Sidmouth observations made since the year 1923.

In the first paper on ϕ Persei, which I communicated to the Society in 1925,* it was shown how the changes in the relative intensities of the components of the bright hydrogen lines could be interpreted in terms of radial velocity measures, the positive velocities corresponding to the cases in which the violet components were stronger than the red, and the negative velocities to those in which the red were stronger than the violet. Thus it becomes possible to convert approximately any radial velocity measures into my estimate curve.

Owing to the peculiarity of the lines in the spectrum of this star, *i.e.* nebulous nature and structure of the hydrogen lines, not many observers have attempted to determine such velocities. Those that I have been able to find are assembled in the following table (Table II.) and converted into my estimate scale.

The greatest recorded value measured was about + 42 kilometres per second, so this has been taken to be equivalent to my eye-estimate of V > > > R, V = R corresponding to "zero" velocity.

An attempt was made to utilize these values to find out which of the two periods, Curtiss's or mine, satisfied them best, when the respective periodicities were projected backwards.

Table II.

Line of Sight Velocity Values for H.D.C. 20336.

Year.	Date.	Km. per sec.	Equivalent Eye-estimate.	Agreement with Curves of		Authority.
				Lockyer.	Curtiss.	
1903	Nov. 17	− 0·9	R > V (L)	..	..	Frost.
1906	Oct. 31	−13·0	R > V	A	..	Barrett.
1907	Oct. 18	+35·0	V > > > R	A	..	Struve.
1908	Jan. 24	−10·4	R > V	..	A	Struve.
1912	Oct.–Dec.	− 3·1	R > V (L)	..	A	Curtiss.
1916	Jan.–Apr.	+41·7	V > > > R	A	A	Curtiss.
1918	Nov.–Dec.	+42·4	V > > > R	..	..	Hnatek.

* *M.N.R.A.S.*, **85**, 584, 1925.

The first four radial velocity values were determined by Frost, Barrett, and Struve,* the next two by Curtiss,† and the last by Hnatek.‡

If the Sidmouth curve with a period of 1568 days be projected backwards to 1903, it picks up only three out of the seven velocity values, as marked in Table II. with the letter A (Agreement).

The best fit that can be made for Curtiss's curve to satisfy my curve from 1925 to 1930 is when his maximum and minimum are 1·5 and 3·5 months respectively late on mine. Projecting his curve of 1710 days period back from those epochs it is found that it also satisfies only three of the velocity values, but these are not all the same as mine as indicated by A in Table II.

While both our periods are of the same order of magnitude, it is impossible for either of them to satisfy such a large change of velocity from + 35·0 km. in 1907 to − 10·4 km. in 1908 in such a short interval of time as 98 days, suggesting a period of about 14 months.

Again, the large positive velocities given by Curtiss in 1916 and Hnatek in 1918, namely, + 41·7 and + 42·4 km./sec. respectively, are only separated by a little under three years, thus representing a period of variation of about this length, which is also much too short.

It will thus be seen that these earlier observations fail to form a check on the accuracy of either of our periods.

Too much weight must not be given to the radial velocity measures, because the character of the lines in the spectrum is by no means conducive to accurate measurement. Frost, Barrett, and Struve confined their measures to absorption lines only, which they described as poor in quality and few in number. When it is remembered that the hydrogen lines are very broad and complicated by the variable bright components in them, that the helium lines and the line of ionised magnesium at λ 4481 are exceedingly nebulous and also broad, the difficulty of measurement becomes obvious. The only well-defined line is [K] of ionised calcium, but this gives a velocity quite different from that of the other lines, as is expected in these stars.

Curtiss measured the hydrogen lines from H_β to H_ζ and the bright components of H_β and H_γ, and while perfectly aware of the difficulty of deriving accurate velocity values, he considered that his means given in Table II. were good representative values.

In the case of Hnatek's two measures, + 38·6 and + 46·2 km. per second, made in November and December of 1918 respectively, the mean of which has been given in Table II., the values for the individual lines "agree well with one another" as he states, and he even regards the difference between the velocities deduced as probably real. If the sign had been negative instead of positive the values would have fairly satisfied both of the periods mentioned above. The Sidmouth observations will be continued throughout the next few years to determine more accurately the length of the period. Since it is known

* *A.J.*, **64**, 1, 1926.
† *P.A.O. Univ. Michigan*, **2**, 29, 1916.
‡ *A.N.*, **210**, 246, 1919.

now that the work will involve the securing of one good photograph every month the labour will not be very arduous.

A point of considerable interest to be noted from the curve of the *wedge intensity measures* (fig. 1) is that the maximum point of the curve in 1927 lies at a much smaller distance above the zero line than the minimum in 1929 lies below this zero line. The reading on the scale is 2·4 in the case of the former and 3·4 in the case of the latter. This fact is the opposite of the result given by the wedge intensity measures in the case of ϕ Persei.* There the maximum was further above the zero line than the minimum was below it, the scale readings being 1·8 and 1·35 respectively. These facts are diagrammatically shown in fig. 2 in the two curves at the top of the figure, that on the left relating to H.D. 20336 and that on the right to ϕ Persei.

To explain the nature of the complicated hydrogen lines it is assumed that the broad hydrogen absorption line has superimposed on it a narrower, but still broad, bright band centrally placed, and in the middle of the latter is a sharp narrow absorption line also centrally placed with regard to the bright band.

This would give the appearance of, say, H_β as having two bright lines symmetrically placed on a broad absorption band. Thus the bright violet component would equal the bright red component in intensity (V = R), and this would represent "zero" radial velocity for the star. If it be assumed that the broad, bright band always maintains its relative position to the broad absorption band, then the movement of the central absorption line would indicate the radial velocity of the star. In fact most radial velocity observers actually measure the position of this narrow absorption line in the spectrum when determining the velocities of stars with this type of spectrum.

When a star reaches its maximum radial velocity of approach the thin absorption line will be moved furthest towards the violet and consequently the bright red component will be very much stronger than the bright violet component. On the other hand, when the maximum radial velocity of recession occurs, then the thin absorption line will be moved furthest towards the red, and so the bright violet component will be much stronger than the bright red component.

Thus it will be seen that the differences in the distances of the maxima and minima points of the curve from the zero line, as pointed out in the curve of intensity differences of H_β in fig. 1, and shown diagrammatically in fig. 2, correspond to the differences between the maximum velocities of approach and recession. These movements of the thin absorption line are diagrammatically shown under the curves in fig. 2.

It is deduced therefore that, in the case of the star H.D. 20336, the maximum radial velocity of approach is less than the maximum radial velocity of recession, while in the case of ϕ Persei the opposite condition occurs.

It is not known whether the spectra of these bright-hydrogen-line stars of Be type are due to two stars moving round their common centre

* *M.N.R.A.S.*, **86**, 484–5, 1926.

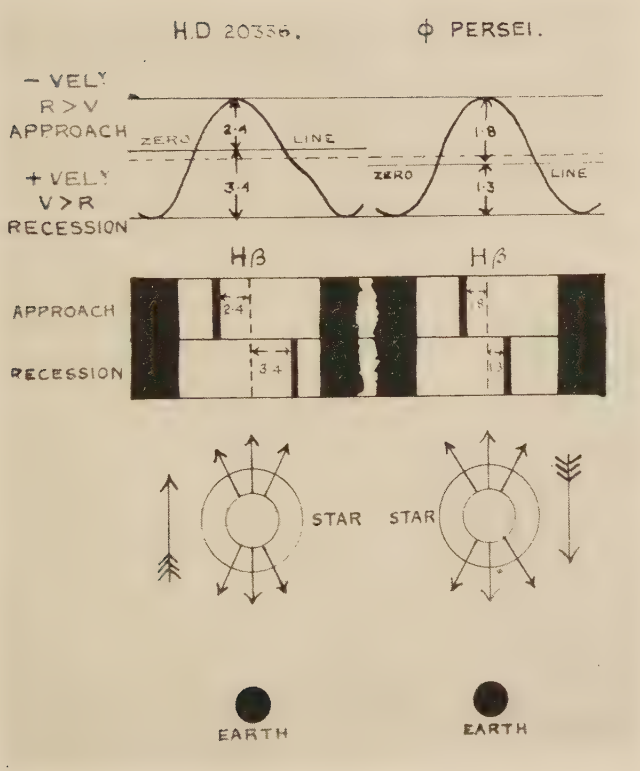

H.D 20336.
φ PERSEI.
– VELY
R > V
APPROACH
ZERO
LINE
2·4
3·4
+ VELY
V > R
RECESSION
1·8
1·3
ZERO
LINE
Hβ
Hβ
APPROACH
RECESSION
2·4
3·4
STAR
STAR
EARTH
EARTH

Fig. 2.

of gravity or to one single star composed of shells round the nucleus. Assuming that the whole spectrum of these stars, together with the complicated structure of the hydrogen lines, is produced by a star of this shell-like nature, as indicated in the lower portion of fig. 2, then when this star is moving towards the earth the maximum velocity of approach will be greater than the maximum velocity of recession, but when it is moving away the maximum velocity of recession will be the greater.

It must be concluded therefore that the star H.D. 20336 is receding from the earth and ϕ Persei is approaching it.

In my earlier paper on this star, mentioned at the beginning of this communication, reference was made to the absorption lines of helium, ionised magnesium and calcium, and also to the bright enhanced lines of iron and titanium, so prominent in the spectrum of ϕ Persei but faint in this star. It was stated there that it was quite probable that when a greater number of photographs had been systematically taken throughout a complete cycle, periodic changes in both the intensity and character of the above-mentioned lines might become evident.

A close examination of all the negatives has, however, failed to discover any changes, cyclical or otherwise, which can be pronounced as certain.

Historical Note.

In my previous communication on the spectrum of this star I gave a brief historical account of the observations, commencing with the statement : " The discovery of the bright-line nature of this spectrum was made by Espin on 1895 September 22, who recorded ' F bright ? '."

Soon after this paper was published I received a letter from him in which he gave a very interesting account of his original observation, together with a photographic copy of the page of his notebook on which the original observation was recorded. The following is an extract from this letter :—

" In 1895 I was, each fine night, sweeping for Red Stars with the $17\frac{1}{4}$ inch. I had also a spectroscope made on the lines suggested by Sir W. Huggins, a direct vision prism before a positive eyepiece. I found out some years before that by moving the prism away from the eyepiece the dispersion was increased, and was only limited by the brightness of the star. On 1895 September 22 I observed 56 stars. The night was noted as very clear. I noticed B.D. + 65°·338 as having F very large. This is in the field with B.D. + 65°·340 (H.D. 20336), differing only $2^m\ 24^s$ in R.A. and 0′·5 in declination. With increasing dispersion, and with the focus made on + 65°·338 it seemed to me that in + 65°·340 there was a faint bright line at the place of β hydrogen. Had it not been the fact of having + 65°·338 to focus on I should probably never have suspected it. It was duly turned up next day in the Draper Catalogue (*H.A.*, 27). It is No. 1848 and is given as Type B. There are additional notes on pp. 143, 280, and it is quite clear that the bright line at this date (1890) had not been recognised. If I remember rightly I then wrote to Professor E. C. Pickering, and

later he answered that the Harvard photographs on close inspection suggested this to be correct. Hence, I presume, it was that in Mrs. Fleming's Catalogue (Vol. 56, No. 6) it was duly noted as having been detected by me."

Summary.

1. Further observations on the spectrum of the bright-hydrogen-line star H.D. 20336 have been made to decide between the claims to accuracy of the two periodicities, namely, 2·3 and 1·4 years, deduced previously by Curtiss and the writer respectively.

2. In the meantime Curtiss has published new values for the period, namely, 1690 and (later) 1710 days.

3. The Sidmouth observations give, by two independent methods of reduction, a period of 1568 days, which, while being of about the same order of magnitude as Curtiss's latest period, is of shorter duration by 142 days.

4. The Sidmouth curve does not conform to that obtained by Curtiss, who described his as approximately a sine curve.

5. An attempt was made to project both periodicities back to see if they satisfied radial velocity values by other observers, but each curve conformed only to three out of the seven available velocity values, only one being common to each curve.

6. It is suggested that some of these radial velocity measures, owing to the complex and nebulous nature of the spectrum lines, are not very satisfactory.

7. From the position of the zero velocity line which represents equality in the intensity of the bright components of the H_β line, it is deduced that the star H.D. 20336 is receding from the earth while ϕ Persei is approaching it.

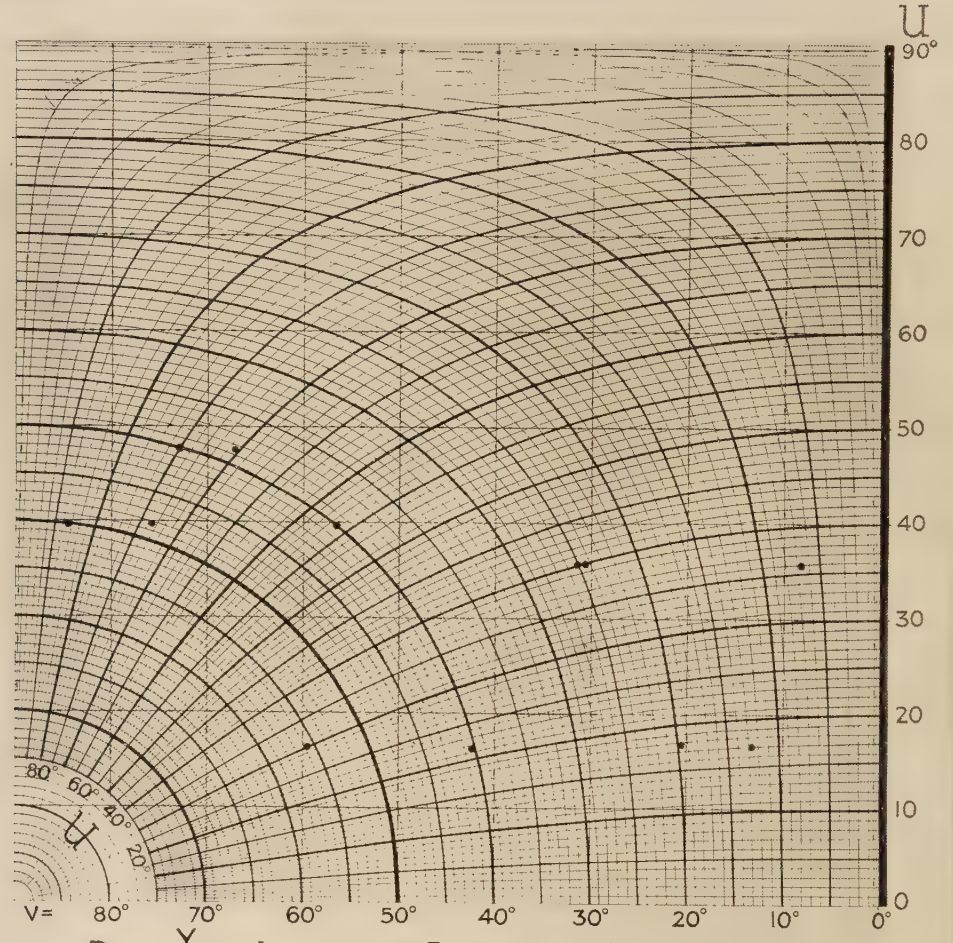
8. Apart from the hydrogen lines, no others, whether dark or bright, can be detected as exhibiting any changes, cyclical or otherwise.

9. Espin's own account of his discovery of bright H_β is given as an interesting piece of history relating to this star.

Graphical Solution of a Spherical Triangle. By L. Becker, Ph.D., Regius Professor of Astronomy in the University of Glasgow. (Plate 8.)

Since this paper was submitted to the Council the author was informed that the method was forestalled in 1905 *by Dr. H. Maurer in a paper entitled "Ueber Aufloesung von Poldreiecks-Aufgaben durch Diagramme, die auf zenitalen Kartenprojektionen beruhen," and accompanied by a chart of curves drawn at intervals of* 5 *degrees as compared with* 1 *degree in the present paper.* (Annalen der Hydrographie, *August* 1905.) *It had attracted no attention.*

Fig. 2.


U
90°
80
70
60
50
40
30
20
10
0
80° 60° 40° 20°
U
V= 80° 70° 60° 50° 40° 30° 20° 10° 0°

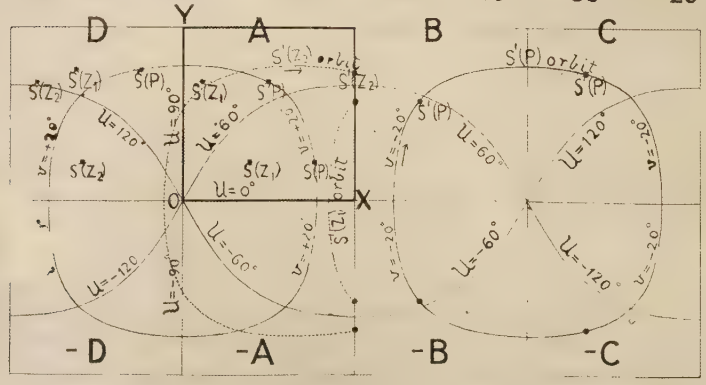

D
Y
A
B
C
S'(Z₂) orbit
S'(Z₁)
S(Z₁)
S(P)
S'(P) orbit
U=120°
U=90°
U=60°
U=0°
U=-60°
U=-120°
V=+20°
V=-20°
X
-D
-A
-B
-C

Fig. 3

1. Consider a right-angled spherical triangle PRS in which the angle at R is 90°; then with the notation shown in fig. 1,

$$\tan x = \cot v \cos \mathrm{U},$$
$$\sin y = \cos v \sin \mathrm{U}.$$

I have calculated the spherical co-ordinates x and y, with an accuracy of 0·05°, for each degree of U and v in the first quadrant. Considering x and y rectangular *linear* co-ordinates of a point S(P), I have drawn graphs of the curves U = const. and v = const., where the constants have successively the values 0°, 1°, 2°, . . . 89°, 89½°, and for v up to 89¾°. These curves are shown on a reduced scale in fig. 2. Those in the other quadrants lie symmetrically to them with reference to the x- and y-axes as exhibited in fig. 3 at the bottom of the plate. The v-curves are reduced to a point at v = 90°, are then almost circular

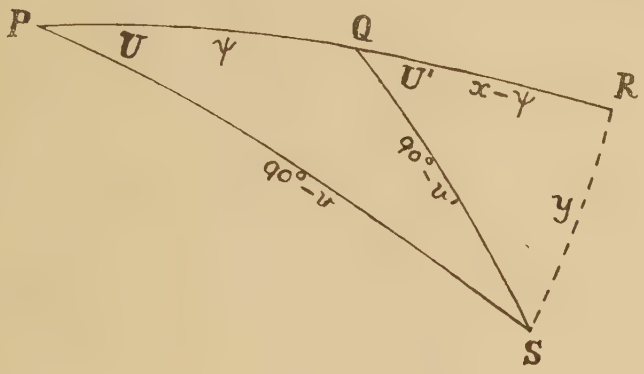


Fig. 1.

round this point, and gradually approach a square at v = 0°. The U-curves start as a straight line at U = 0°, then become a sine-curve which gradually changes, at U = 90°, into straight lines intersecting at right angles.

The branches of the curves in the lower half of fig. 3, which belong to the third and fourth quadrants of U, are the reversals of those in the upper half which represent the first and second quadrants of U. The lower half may be dispensed with if U in the last two quadrants be counted negative, *i.e.* expressed between − 180° and 0°.

Each section A, B, C, D is a reflected image of its neighbouring sections, the reflections occurring either on the zero line of v (where v changes its sign) or on the 90°-line of U (where U changes from the first to the second quadrant). It is therefore unnecessary to draw the curves for each section A, B, C, D, those of section A as given in fig. 2 being adaptable to the other sections, with fig. 3 as a guide.

2. In a spherical triangle PQS the point S has, with reference to pole P, the spherical co-ordinates x, y and the polar co-ordinates U, v.

The corresponding co-ordinates with reference to pole Q are $x - \psi$, y, and U′, v'. Thus on the diagram of the U- and v-curves the point S(P) lies at the intersection of the curves (U, v), and S(Q) at the intersection of the curves (U′, v'). There is this remarkable property of the two points S(P) and S(Q) that they lie on a line parallel to the x-axis and at mutual distance ψ. One being given, the other can be found. Hence if U, v, ψ of the spherical triangle be known, the diagram gives U′, v', and *vice versa*.

3. *Example.*—Variation of azimuth and altitude of S in the course of the day. Let P designate the Pole of the Heavens and Z(= Q) the Zenith. The co-ordinates of S referred to P are hour-angle, t, and declination, δ, and those referred to Z are azimuth, A, and altitude, h. S(P) lies at the intersection of curves U = t, $v = \delta$, and S(Z) lies at the intersection of curves U′ = A, $v' = h$. S(P) moves in the course of the day on the v(= δ)-curve. S(Z) moves in a similar curve obtained by transposing this v-curve in the x-direction through $90 - \phi$, the co-latitude. Fig. 3 shows the paths of S(Z) in two cases, for geographical latitudes + 56° and − 30° (dotted curves). At each moment, t, the point S(Z) lies at the intersection of two curves U′, v', which determine the azimuth and altitude. The curve is not drawn in section B, which corresponds to S below the horizon.

4. *Application to Astronomical Problems.*—(A) Conversion of R.A. and Decl. into co-ordinates of other systems. In the table below, the pole Q designates successively (1) the pole of the ecliptic, (2) the pole of the galaxy, and (3) the apex of motion of the solar system. The assumed values of the R.A. and Decl. of the pole of the galaxy are 190° and + 28°, and those of the apex 270° and + 28° respectively. S(Q) lies to the left of S(P) at the distance ψ given in the table.

Point.	Co-ordinates.		Pole of Ecliptic.			Pole of Galaxy.			Apex.		
S	R.A.	Decl.	50°·0	+25°·0		50°·0	+25°·0		310°·0	−25°·0	
S(P)	U,	v	180°−40°·0	+25°·0	D	−(180°−40°·0)	+25°·0	−D	40°·0	−25°·0	B
	ψ		23°·4			62°·0			62°·0		
S(Q)	U′,	v'	180°−35°·9	+ 6°·5	D	−(180°−39°·8)	−24°·5	−C	39°·8	+24°·5	A
S	l,	b	54°·1	+ 6°·5		−140°·2	−24°·5		+39°·8	+24°·5	
Formulæ :			U=R.A.−270°			U=R.A.−190°			U=R.A.−270°		
			v=Decl.			v=Decl.			v=Decl.		
			l=U′+270°			l=U′			l=U′		
			b=v'			b=v'			b=v'		

(B) Find the parallactic angle between great circles SP and SZ. The celestial object, S, stands at hour-angle t and decl. δ and its altitude is h. Consider the co-ordinates of zenith, Z, relatively to poles P and S. Z(P) has the co-ordinates U = t, $v = \phi$, and Z(S) has the co-ordinates U′ = (180° − q), $v' = h$.

Example.—t = 30°·0, δ = + 12°·0, ϕ = + 55°·9. The co-ordinates of Z(P) are U = 30°·0, v = 55°·9, and Z(P) lies in section A.

78°·0 towards the left lies Z(S) in section D, at intersection of U′ = 180° − 21°·5, and v' = + 40°·3. Hence q = 21°·5, h = 40°·3.

5. *Nautical Problems.* (A) *Position-line.*—The R.A. and Decl. of the observed object being given, the hour-angle, t_0, at a given Greenwich mean time may be calculated for a place whose geographical longitude and latitude are λ_0, ϕ_0. The point S(P) lies at the intersection of the curves U = t_0, v = Decl. At distance $\psi = (90° - \phi_0)$, to the left, lies S(Z). The curves U′ and v' intersect in S(Z), so that A_0 = U′, $h_0 = v'$.

The observed altitude being h at the above time, $(h - h_0)$ and A_0 determine the position-line in the usual way.

Example.—Hour-angle = − 17°·8, Decl. = − 19°·7, latitude ϕ_0 = + 56°·0. The co-ordinates of S(P) are U = + 17°·8, v = − 19°·7.

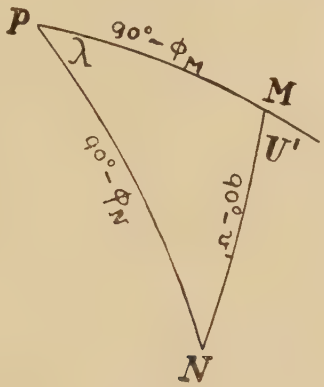


Fig. 4.

S(Z) lies 34°·0 to the left where U′ = 17°·1 and v' = + 12°·8 intersect. Hence A_0 = − 17°·1 and h_0 = 12°·8.

(B) In a partly clouded sky an observed star may not have been identified. In such a case the approximate azimuth, A, ought to be measured with the gyro-compass. S(Z) lies at the intersection of U′ = A, v' = h. To the right, at distance $(90° - \phi_0)$, lies S(P) where (U, v) intersect. U being the approximate hour-angle at the observed Greenwich mean time, T, the R.A. of the star has the value = T − 12^h + R.A.M.S. − λ_0 − U. This R.A. and Decl. (= v) allow the star to be identified. The observation is then treated as under (A).

(C) *Great Circle Sailing* (fig. 4).—Find for a given place, M, the azimuth of the great circle passing through another given place, N. Let λ designate the longitude of N relatively to M, and ϕ_M and ϕ_N the geographical latitudes. N(P) lies at the intersection of U = λ and $v = \phi_N$. N(M) lies $(90° - \phi_M)$ from it to the left and here intersect the curves (U′, v'). The desired azimuth of the great circle is U′, and the distance between the two places is $(90° - v')$ expressed in degrees. During a journey the point N(P) travels on the same curve, $v = \phi_N$.

Example.—Place M lies at latitude + 50°·0 and place N at 40°·0 ; the relative longitude is 75°·0. The co-ordinates of N(P) are U = 75°·0, v = 40°·0. N(M) lies 40°·0 to the left where U′ = (180° − 70°·5) = 109°·5 and v' = 38°·3 intersect. Hence the azimuth of the great circle at M is 109°·5, and the distance MN is 51°·7.

(D) *Find the Half-way Place* (fig. 5).—Let M and N be the given places and X the half-way place. Choose PX as reference meridian. M(P) lies on the v(= ϕ_M)-curve and N(P) lies on the v(= ϕ_N)-curve. These points must be so chosen that (1) they have the same y-co-ordinate, and (2) N(P) must lie on the U_N(= $\lambda - U_M$)-curve when M(P) lies on the U_M-curve. These points being found, U_M is read off,

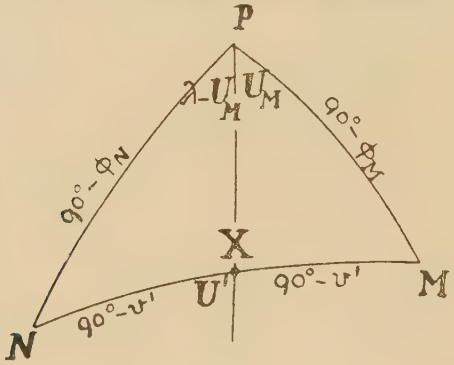


Fig. 5.

and their x-co-ordinates x_M and x_N are measured. The four points M(X), N(X), M(P), N(P) lie on a line parallel to the x-axis. N(X) has the x-co-ordinate $\frac{1}{2}(x_N - x_M)$ and in it intersect the curves U′, V.′ The readings U_M, x_M, x_N, U′, v' determine the result. The half-way place, X, has the longitude U_M relatively to M and latitude $(90° - \frac{1}{2}(x_M + x_N))$. The distance from X to M or N is $(90° - v')$ degrees. At X the direction to N has azimuth U′.

Example.—ϕ_M = 50°·0, ϕ_N = 40°·0, λ = 140°·0. M(P) lies on the v(= 50°·0)-curve, N(P) on the v(= 40°·0)-curve. Both have the same y-co-ordinate for U_M = 83°·4. Further, U_N = 56°·4, x_M = 5°·5, x_N = 33°·4. N(X) lies on the same line at x-co-ordinate 14°·0 and here intersect U′ = 73°·7 and v' = 48°·2. The half-way place has longitude 83°·4 and latitude = 70°·5. The distance MX = NX = 41°·8 and at X the great circle MN intersects at azimuth + 73°·7.

Discussion of Observations of Three Long-period Variables, made by A. N. Brown, M.A. No. 3, U Persei, 1909 September–1930 March. By Mary A. Blagg.

[These observations, down to 1929 November, have been published in detail in *M.N.* in four papers: 1911 April, 1919 May, 1924 June, and 1929 December. In these papers Hagen's magnitudes were adopted for the comparison stars, but for the purpose of this discussion the observer replaced them by those of Harvard.]

The discussion of these observations of U Persei, the last of this series of three stars, has not yielded results either so interesting or so suggestive as in the other two cases. This seems to be mainly due to the greater length of the period (about 320 days), which involves a smaller number of cycles for investigation in the $20\frac{1}{2}$ years, and to a greater apparent irregularity in the variations of the curve. It is more obvious even than with the others that no dates for maximum or minimum can be fixed with anything like precision. Even when they may *seem* to lie almost certainly within a limit of plus or minus 2 or 3 days, this is often found on examination to rest upon one or two observations only. When these epochs are found by analysis, instead of being judged by the appearance of the observations round about the epochs, the precision obtained is little greater, if indeed it be even equal in cases of great irregularity in the curve, depending as it does for accuracy on the curve of the particular cycle being pretty regularly observed, and on its form not greatly differing from the average one.

It is, however, less difficult to find approximately the average length of period during a given series of consecutive cycles. Inspection suggested a period of 320 days, and the observations were divided into cycles of that length, subdivided into 12 portions of $26\frac{2}{3}$ days each. This arrangement gave between 23 and 24 cycles. The mean magnitudes for each portion, when taken separately for the first and second halves of the series, gave no "correction to period," and the provisionally assumed average period of 320 days was retained for the purposes of discussion.

As often happens where there is a large number of cycles, the "mean curve" obtained by the process described above (using 22 consecutive cycles) was very smooth, in spite of the irregularities in the individual cycles. The actual figures are:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
8·25	7·77	7·72	8·11	8·48	8·96	9·33	9·77	10·22	10·33	9·91	9·13

Analysis gives the formula:

$$1{\cdot}24 \cos(\theta - 244^\circ{\cdot}5) + {\cdot}24 \cos(2\theta - 215^\circ{\cdot}6).$$

(The 3θ term is too small to be significant.)

This form, with the mean mag. 9·0, works out as follows, showing very close agreement:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
8·27	7·76	7·74	8·08	8·52	8·92	9·33	9·80	10·22	10·32	9·92	9·12

The interval from maximum to minimum shown here is about 189 days, and that from minimum to maximum about 131 days—a greater proportional difference than was found for the other two stars, a circumstance connected with the much higher proportion borne by the coefficient of cos 2θ to that of cos θ. (For RT Cyg. the figures were as 1 to 38; for V Cas., as 1 to 55; while here they are as 1 to $5\frac{1}{6}$.) The incidence of the phases is, however, nearly alike in all three. Correcting the phase in cos θ to $-180°$ in each, we get for cos 2θ the phases $-98·8$, $-97·1$, and $-86·6$ respectively (see *M.N.*, **90,** 5, 554, 1930). For the present star these intervals, as also the lengths of individual periods, seem to vary considerably; but the extreme variations given in the papers previously mentioned may be to some extent illusory, owing to the possibility of large accidental errors in most of the dates concerned.

As with the other two variables, each observation in each cycle was compared with its position in the mean curve, and the residuals in magnitude thus obtained were tabulated according to months, in order to ascertain whether there seemed to be evidence of "seasonal" effects. For this purpose the series was divided into two, and also into three, sections chronologically. The mean residuals for January and February were negative (= relative brightness) in each of the five divisions, and in the whole were respectively $-·12$ and $-·20$ mag. —this in spite of a time of great relative faintness from January to March in 1918. From September to December (inclusive) the mean residuals were positive in each division, and in the whole were respectively $+·13$, $+·12$, $+·06$, and $+·04$ mag. For the remaining six months the results were inconsistent. These figures suggest a possible real seasonal effect in the autumn and winter half of the year. The negative residuals for February, and the positive residuals for November and December, are found in connection with all the three variables discussed in this series of papers, in greater or less degree; but in other months there is much divergence. Obviously, the relations between orientation and season would differ greatly for the three stars, and in this instance the question is complicated by the near approach of the star's period to a year in length.

The following table gives details of sudden brief spells of relative brightness or faintness, exceeding a difference of $\pm 0·75$ m. from the mean curve, obtained (as explained) with the assumed mean period of 320 days.

The observations are in most cases so few that it would be unwise to attempt to draw conclusions as to any possible recurrences of such phenomena, even were there definite apparent indications of such recurrences. Two points, however, may be worthy of note: (1) A serious error in the assumed mean length of period would produce, for a given portion of the curve, residuals of *opposite* sign in the early and late cycles to a marked degree. This is not appreciably the case. (2) The months September to December do not appear at all in (*b*), and those of January and February appear in both (*a*) and (*b*), so that the average faintness and brightness found above for these months

respectively cannot be due to these sudden, apparently abnormal occasions.

TABLE I.

(a) *Relative Brightness.*

Date.	Cycle.	Portions of Curve.	Mean Resid.	No. Obs.
			Mag.	
1909–10 Dec., Jan.	1	5, 6	−0·80	3
1914 Feb.	6	1, 2	−0·85	4
1916 Feb.	8	4, 5	−0·90	2
„ Aug.	8	11, 12	−0·87	3
„ Sept.	9	1	−1·05	2
1921 Jan., Feb.	14	1, 2	−0·87	4
1924 May to July	17	10–12	−1·00	7
1929 Mar., Apr.	23	4, 5	−1·02	5

(b) *Relative Faintness.*

Date.	Cycle.	Portions of Curve.	Mean. Resid.	No. Obs.
			Mag.	
1913 Jan., Feb.	4	11	+0·80	3
1918 Feb., Mar.	10	8, 9	+1·00	4
1921 Mar., Apr.	14	3	+0·80	3
1926 May	20	1	+0·85	2
1927 Mar., Apr.	21	1	+0·85	2

The residuals in magnitude were also used to obtain by analysis corrections to "epoch" in each cycle, taking part of each cycle for the maximum and part for the minimum. It has been already stated that the results show a smaller range of variation than is given by the estimated dates in the published papers mentioned above, but that they, too, are extremely uncertain in most cases.

Taken at their face value, they show both for maxima and minima a marked fluctuation in position. The sign is positive for the first few cycles; it then becomes negative with a few exceptions, until we come to the last five or six, which again have the positive sign. This is so both for maxima and minima, though the changes of sign do not occur at the same points for each. No trustworthy conclusions can be drawn from an apparent fluctuation so nearly coinciding with the length of the series, especially in view of the uncertainty of many of the figures, but the movement is too definite and too consistent to be purely accidental. The actual figures depend, of course, in detail upon the assumed mean length of period, but the dates thus obtained for maxima and minima, and the individual lengths of periods deduced, are independent of that assumption.

Table II. gives the average value found for the corrections to epoch, taken in groups of consecutive cycles, and Table III. the average value for the deduced lengths of periods, similarly treated. "M" stands for those taken from maxima, and "*m*" for those from minima. The number of each epoch is the same as in the observer's published papers. The 3rd and 8th maxima are not used, the results for them being totally untrustworthy owing to extremely long gaps in the observations.

(If the results for maxima 3 and 8 be admitted, the negative residual for the total mean vanishes.)

The apparent fluctuation already mentioned can be clearly seen in each of these three groupings, but shorter fluctuations, of which there

are signs in the corrections taken one by one, are here masked through the grouping.

TABLE II.

Corrections to Epoch.				Corrections to Epoch.			
Epochs.	M.	*m*.	Mean.	Epochs.	M.	*m*.	Mean.
1 to 4	+ 9·5	+ 6·55	+ 8·0	1 to 5	+ 6·4	+ 5·8	+ 6·1
5 ,, 8	− 8·5	−10·3	− 9·4	6 ,, 11	− 8·7	−12·8	−10·7
9 ,, 12	−13·8	− 7·9	−10·8	12 ,, 17	−11·8	+ 4·6	− 3·6
13 ,, 16	−13·25	+ 6·5	− 3·4	18 ,, 23	+ 8·7	+ 5·85	+ 7·3
17 ,, 20	+ 2·45	+ 2·0	+ 2·2	1 to 4	+ 9·5	+ 6·55	+ 8·0
21 ,, 23	+16·5	+ 4·9	+10·7	5 ,, 15	−13·6	− 4·2	− 8·9
Total	− 1·5	+ 0·4	− 0·5	16 ,, 23	+ 9·4	+ 3·8	+ 6·6

The general expression for the maximum would be $2418294 + 320n + F_s$; and for the minimum, $2418483 + 320n + F_s$; where F_s stands for the sum of fluctuations, and the initial dates are approximate only.

The means of "M" and "*m*" given in Tables II. and III. have possibly little significance, owing to the apparent independence in the fluctuations of each. Separate analysis gives a variation of 13 + days in epoch each way for both (in from 22 to 24 cycles), but the fluctuation appears to have different culminating points for each. This may explain the complicated variations in the lengths of M–*m* and *m*–M.

Cycle 1 is the length from Epoch 1 to Epoch 2, and so on, Cycle 22 being that from Epoch 22 to Epoch 23. Asterisks (*) mark numbers which differ largely from those to be obtained from the estimated dates in the published papers, and brackets show those which differ *in sign* from the latter. In the four instances of brackets in column 4, the actual difference is very small. In the "13 to 17" line, in spite of much larger numbers in columns 2 and 3, the observer's *mean* is + 2·0 days—almost identical with that in the table.

When the individual figures for the deduced lengths of period are analysed for fluctuations of 22 to 24 periods, taking maxima and minima *separately*, we obtain a fluctuation of about 5 days each way for each, but the position of the phases differs by something like 90°, instead of by 7° or 8°, the equivalent (roughly) of half a cycle. In keeping with this, the two parts of the cycle, M–*m* and *m*–M, do not vary either directly or progressively with one another. On the contrary, when M–*m* is *shorter* than the average, the *m*–M which follows is *longer* in 9 cases out of 10, and when M–*m* is *longer*, the following *m*–M is *shorter* in 11 cases out of 12; while the *preceding* *m* M differs in sign in 17 out of 22 cases. But the two parts of the cycle do not vary actually inversely with each other—which would make the length of the full period almost stationary. When analysed, each of the two parts shows separately a systematic fluctuation (in about 22 to 24 cycles) of over 13 days each way, with a difference in

TABLE III.

Lengths of Periods: Simple Group Averages (320 days +).

5 Cycles.	M. 320+	*m.* 320+	Mean.
1 to 5	−4·7	(−3·9)	−4·3
2 „ 6	−5·3	−4·0	−4·65
3 „ 7	..	−5·2	− ?
4 „ 8	−3·1	−4·5	−3·8
5 „ 9	0·0	−2·6	−1·3
6 „ 10	+0·2	+1·6	+0·9
7 „ 11	−3·3	+3·1	(−0·1)
8 „ 12	..	+5·3	+ ?
9 „ 13	−3·1	+5·2	+1·0
10 „ 14	(−1·0)	+3·6	+1·3
11 „ 15	+5·6	+2·6	+4·1
12 „ 16	+6·5	−2·7	+1·9
13 „ 17	+6·0 *	−1·6 *	+2·2
14 „ 18	(+5·3)	(−0·1)	+2·7
15 „ 19	(+2·9)	(+0·5)	(+1·7)
16 „ 20	−1·0	−0·6	−0·8
17 „ 21	(+1·3)	+4·1	+2·7
18 „ 22	(+5·0)	0·0	+2·5
19 „ 23	+3·4	..	..

11 Cycles.	M. 320+	*m.* 320+	Mean.
1 to 11	−3·2	(−0·5)	−1·85
2 „ 12	−4·4	+0·3	−2·05
3 „ 13	−3·9	0·0	−1·95
4 „ 14	−0·8	+0·3	(−0·25)
5 „ 15	+1·7	+0·5	+1·1
6 „ 16	+1·9	(+0·1)	+1·0
7 „ 17	+0·8	(+1·4)	+1·1
8 „ 18	..	+2·3	..
9 „ 19	+1·9	+2·6	+2·25
10 „ 20	(+1·3)	+1·3	+1·3
11 „ 21	+2·4	(+1·2)	+1·8
12 „ 22	+4·5	0·0	+2·25
13 „ 23	+4·0	..	..
7 Cycles.			
2 to 8	−4·6	−3·3	−3·95
9 „ 15	+4·3	+3·6	+3·95
16 „ 22	+1·3	−0·9	(+0·2)
2 to 10	−3·3	−1·2	−2·25
11 „ 21	+2·4	+1·2	+1·8
2 to 21	−0·2	+0·1	−0·05

The averages from the observer's estimates are: Cycles 2 to 21, M = 319·1, *m* = 320·5, thus differing very little from the above.

phase of about 184°, agreeing with the near approach to an inverse ratio of variation. We may fairly conclude that the fluctuations in both have the same cause and the same length of "period." A secondary variation in the star's magnitude, smaller in amount than, and in length nearly (but not exactly) double or half that of, the principal variation, might produce some such effect.* Indications of a fluctuation in phase in a little over twice the main average period were found at an early stage of the investigation, which would fit in with this, and assumed fluctuations of this length and of ten times this length together much reduce the sum of the residuals. But the possible errors are often as great as the fluctuation figures, and the data are quite inadequate for testing any such theories, which are therefore thrown out merely in the hope of their possibly suggesting lines of investigation to future workers on this star. A salient and probably significant point is that (by analysis) each part of the cycle (M–*m* or *m*–M)

* A fluctuation in length of the cos 2θ part of the star's variation would not meet the case.

fluctuates in length (during 22 or rather more full cycles) at least $2\frac{1}{2}$ times as greatly as does the complete cycle (M–M or *m*–*m*). It is also pretty certain that these are not the only fluctuations, but that other shorter ones increase to a considerable extent the total variation in the length of period.

Table IV. gives details of such dates, etc., as are comparatively satisfactory. Cases are excluded where the date from analysis differs by more than 8 days from the observer's estimate, or there is a gap of over 20 days in the observations near the epoch in question. The dates from analysis are subject to a possible *constant* error of about 2 days in the reading of the positions in the mean curve. This does not affect the lengths of periods.

TABLE IV.

No. of Epoch.	J.D. of Maximum.		J.D. of Minimum.		Deduced Average Lengths of Period.			
	By Analysis.	Observer's Difference.	By Analysis.	Observer's Difference.	Epochs.	By Max. 320+	By Min. 320+	No. of Cycles.
		d.		d.				
2	2418951·7	+7	2419128·7	−5	2 to 10	−2·6	−2·0	8
4	..	..	9767·7	+2	2 „ 12	−4·3	..	10
9	2421159·4	−3	..	..	4 „ 20	..	+0·4	16
10	1490·8	−3	2421673·1	−3	4 „ 22	..	+0·3	18
11	1801·8	−1	..	..	9 „ 14	−3·1	..	5
12	2108·9	+8	..	..	11 „ 16	+5·6	..	5
14	2744·1	−5	..	..	10 „ 17	..	−0·3	7
16	3429·8	−4	..	..	10 „ 22	+1·4	+1·5	12
17	..	..	3910·7	−2	14 „ 22	+5·4	..	8
20	..	..	4893·8	+4	17 „ 23	..	+2·3	6
22	5347·7	−7	5531·4	+3	2 „ 22	−0·2	+0·1	20
23	..	..	5844·5	+3				

"*Epoch* 2 to 22" has the same meaning as "*Cycles* 2 to 21" inclusive (*cf.* Tables III. and IV.).

When the lengths of periods deduced from analysis are taken individually, omitting only those derived from the 8th maximum, it is found that periods of less than 320 days prevail with very few exceptions down to the 13th cycle by maxima and to the 9th by minima. After this, periods of over 320 days predominate for both, but with more exceptions, which include the *last* deduced period for each. This suggests the possibility of a small secular lengthening of the star's period; but against that hypothesis is the fact that the apparently longest periods occur some time before the end, and the apparently shortest ones some time after the beginning, of the series. Thus, the apparent fluctuation in 22 to 24 cycles seems the more likely explanation. It is, however, possible that a very small secular lengthening may exist in addition to various fluctuations.

A further 10 years' careful observation of this star should provide

sufficient data for testing the suggestions thrown out in this paper to the extent of proving or disproving their *probability*; but at least 20 years more would be needful for a thorough testing of the reality of the apparent long fluctuations. Any long series of earlier observations might serve the same purpose.

Summary.

1. A series of 882 observations of U Persei, made by A. N. Brown, extending over more than 20 years, have been tabulated and discussed.

2. A mean length of period of 320 days was found to suit the observations, and a smooth mean curve of variation was deduced, resembling those of the two stars previously discussed, but with the rise more rapid than the fall to a still greater degree. The full period and also the parts of it from maximum to minimum and from minimum to maximum showed greater variations in length, of an apparently more irregular character, than the periods of the other stars.

3. There were a certain number of great and sudden changes of magnitude, with no obvious regularity in their occurrence. Some small indications of seasonal effects in the months September to February were found: as regards November, December, and February, these agree in sign with the findings for RT Cyg. and V Cas.

4. Corrections to phase (for maximum and minimum separately) were worked out by analysis in each cycle, and the corresponding individual lengths of period deduced; but the individual results being for the most part very doubtful, only group-averages are given in the tables.

5. Amidst the general appearance of irregularity in the variations, there are certain indications of underlying systematic fluctuations, in particular one of from 22 to 24 full cycles in length. This being just about the length of this series, it is impossible effectively to test its actual details, or even its certain existence. This apparent fluctuation is found by analysis, separately, in the corrections to phase, M and *m*; and in the deduced lengths of M to M, *m* to *m*, M to *m*, and *m* to M; and it comes out *greater* in the last two than in the full cycles. But the phases of culmination, etc., differ largely in all cases one from another. From these notes some future worker on the variation of the star may possibly be helped to find the true explanation.

Micrometrical Measures of Double Stars.

By Rev. T. E. R. Phillips, M.A.

The following measures have been made with the 8-in. refractor lent by the Society :—

Star.	1925. R.A.	1925. Dec.	Mags.	Position Angle.	Distance.	No. of Nights.	Date. 1900+
	h m	° ′		°	″		
OΣ 12 (λ Cas)	0 27·6	+54 7	5·6, 5·9	164·4	0·48	3	29·79
Σ 59	0 43·6	+51 2	7·2, 8·1	146·9	2·19	3	29·79
Σ 60 (η Cas)	0 44·4	+57 26	4·0, 7·6	270·4	8·16	3	29·67
				271·6	8·38	3	30·76
Σ 73 (36 And)	0 50·9	+23 14	6·2, 6·8	80·5	0·55	3	29·79
Σ 162	1 44·5	+47 32	7·2, 7·5	212·2	2·02	3	29·79
Σ 178	1 48·0	+10 26	7·8, 7·8	200·2	3·31	3	30·79
Σ 186	1 52·0	+ 1 28	7·2, 7·2	43·0	1·30	3	29·80
OΣ 38 (γ^2 And)	1 59·3	+41 58	5·0, 6·2	105·1	0·51	3	29·67
Σ 333 (ε Ari)	2 54·9	+21 3	5·7, 6·0	204·2	1·56	3	29·79
Σ 572	4 33·9	+26 48	6·5, 6·5	196·9	3·73	3	30·77
Σ 577	4 37·1	+37 23	7·7, 7·7	54·4	1·26	3	30·76
Σ 644	5 5·2	+37 13	6·7, 7·0	220·7	1·57	4	30·76
Σ 716 (118 Tau)	5 24·7	+25 6	5·8, 6·6	203·4	4·79	5	29·24
OΣ 545 (θ Aur)	5 54·6	+37 13	3·0, 7·5	330·1	2·91	3	30·85
Σ 941	6 33·4	+41 39	7·0, 8·0	79·4	1·70	3	30·77
Σ 1110 (Castor)	7 29·8	+32 3	2·7, 3·7	211·1	4·38	6	28·16
				210·5	4·32	6	29·24
				208·6	4·26	3	30·85
Σ 1196 (ζ Cnc)	8 7·9	+17 53	5·0, 5·7	204·6	0·48	6	28·22
				193·1	0·47	5	29·24
Σ 1291	8 49·7	+30 52	5·9, 6·4	319·2	1·51	3	30·37
Σ 1356 (ω Leo)	9 24·4	+ 9 23	5·9, 6·7	134·2	0·86	3	29·25
Σ 1424 (γ Leo)	10 15·8	+20 13	2·0, 3·5	117·7	3·81	5	28·28
Σ 1523 (ξUMa)	11 14·2	+31 58	4·0, 4·9	76·5	1·86	3	28·21
				66·0	1·48	5	29·28
				46·0	1·09	4	30·83
Σ 1536 (ι Leo)	11 20·0	+10 57	3·9, 7·1	24·2	1·18	4	28·21
Σ 1670 (γ Vir)	12 37·9	− 1 2	3·0, 3·0	320·3	5·89	4	28·37
Σ 1728 (42 Com)	13 6·4	+17 55	6·0, 6·0	9·6	0·51	4	28·32
Σ 1865 (ζ Boo)	14 37·6	+14 3	3·5, 3·9	135·3	1·04	5	28·24
				134·6	1·06	4	29·26
				132·6	1·13	3	30·58

Star.	1925. R.A.	1925. Dec.	Mags.	Position Angle.	Dis. tance.	No. of Nights.	Date. 1900+
	h m	° ′		°	″		
Σ 1888 (ξ Boo)	14 47·9	+19 24	4·7, 6·6	31·4	3·40	5	28·34
				28·7	3·72	5	29·26
				24·5	3·81	3	30·51
Σ 1932 (1 CrB)	15 15·1	+27 6	6·5, 6·8	38·4	0·47	5	29·33
Σ 1937 (η CrB)	15 20·1	+30 33	5·2, 5·7	187·5	0·54	5	29·37
Σ 2055 (λ Oph)	16 27·1	+ 2 8	4·0, 6·1	104·2	0·68	3	28·52
Σ 2084 (ζ Her)	16 38·5	+31 43	3·0, 6·5	44·0	0·96	3	28·60
Σ 2130 (μ Dra)	17 3·8	+54 34	5·0, 5·1	113·8	2·29	3	29·79
Σ 2140 (α Her)	17 11·2	+14 28	3·0, 6·1	111·1	4·44	3	29·79
Σ 2272 (70 Oph)	18 1·7	+ 2 31	4·3, 6·0	124·2	6·18	4	28·49
				122·9	6·31	6	29·53
				121·4	6·70	3	30·67
Σ 2382 (ε′ Lyr)	18 41·9	+39 35	5·1, 6·0	6·0	2·87	5	28·33
Σ 2383 (ε² Lyr)	18 41·9	+39 32	5·1, 5·4	114·9	2·30	5	28·33
Σ 2525	19 23·5	+27 10	7·4, 7·6	300·3	1·17	4	30·57
Σ 2579 (δ Cyg)	19 42·6	+44 56	3·0, 7·9	272·7	1·59	6	29·56
OΣ 413 (λ Cyg)	20 44·5	+36 13	5·0, 6·3	42·7	0·65	5	29·77
Σ 2732	20 46·4	+51 37	6·7, 8·7	72·3	3·89	3	30·57
β 155	20 48·7	+51 8	6·5, 7·4	29·5	0·78	3	30·57
OΣ 416	20 49·2	+42 26	7·8, 8·1	128·6	8·09	3	30·58
OΣ 418	20 51·8	+32 25	7·3, 7·5	286·8	1·37	3	30·57
OΣ 431	21 8·6	+40 56	7·6, 8·0	119·8	3·18	3	30·68
OΣ 432	21 11·4	+40 50	6·8, 7·2	121·7	1·25	3	30·68
OΣ 437	21 17·7	+32 8	6·5, 7·2	33·5	1·98	4	30·61
OΣ 456	21 52·7	+52 10	7·8, 8·0	30·6	1·28	3	30·57
Σ 2912	22 26·2	+ 4 3	5·8, 7·2	114·8	0·55	4	30·74
OΣ 494	23 17·1	+21 33	7·4, 8·1	80·2	3·39	3	30·69

Headley Rectory, Epsom :
1930 *December.*

A Solar Eruption on 1930 *November* 25.

(*Communicated by the Astronomer Royal.*)

1. A solar eruption of considerable magnitude near the centre of the sun's disc was observed between $10^h\ 34^m$ and $10^h\ 58^m$ G.C.T. on 1930 November 25 with the spectrohelioscope at the Royal Observatory, Greenwich. The eruption comprised the contemporary appearance of brilliant patches of hydrogen flocculi with little or no radial

velocity and rapidly changing dark markings moving with high radial velocities, increasing to about 450 km./sec. The observed phenomena (seen in the light of Hα) would appear to represent the end-on aspect of a highly eruptive prominence being shot out from the chromosphere.

2. The eruption took place on the sun's central meridian about 9° south of the centre of the disc. The heliographic position was longitude 126°, latitude 7° S. It followed a sunspot surrounded by fairly bright flocculi, the position of the spot being longitude 135°, latitude 6° S., and its area 200 millionths of the sun's hemisphere.

3. Observation of the disturbed region was first made at $9^h\ 51^m$, when a narrow dark ∫-shaped marking was present. Measures of radial velocity along its length showed the greater part of it to be stationary, but the extreme northern tip, curving round the top of the bright flocculi surrounding the sunspot, to be descending with a velocity of 27 km./sec. At $10^h\ 34^m$ there were distinct signs of change; the marking had become broader and blacker, and a few points of brilliant flocculi had appeared near its edges about the middle of its length. On turning the line-shifter towards the violet the dark marking was plainest at a reading equivalent to an approaching velocity of 38 km./sec. The subsequent observations are as follows (+ denoting inward velocities to the sun and − outward velocities):—

$10^h\ 36^m$. Measure of velocity − 40 km./sec.

$10^h\ 39^m$. In the course of sweeping with the line-shifter a compact dark marking became visible on the extreme violet side of the zero, which was about the middle of the line-shifter scale. The zero was therefore set at the extreme red end, but it seemed that the maximum density of the marking had not been reached when the line-shifter was turned to the extreme violet end, a setting that corresponded to a radial velocity of − 380 km./sec.

$10^h\ 42^m$. A pair of brilliant patches of flocculi were noted in the field, but attention was concentrated on the dark markings on account of their large motions. Three distinct and differing aspects of the dark marking were seen at readings corresponding to − 190, − 380, and − 450 km./sec. The last reading, which exceeded the range of the line-shifter, was obtained by turning the slow motion of the grating until the dark marking appeared. The rotating prisms * were then stopped, and the spectrum contiguous to Hα was viewed through one of the small windows constructed for this purpose at the top and bottom of the slit respectively. It was seen that the atmospheric line λ6553, *i.e.* 10A to the violet of Hα, was practically in alignment with the slit, this displacement being equivalent to a velocity of about − 450 km./sec. At this wave-length the marking was quite distinct though not of intense blackness, its length being from 1′ to 2′.

$10^h\ 47^m$. Remarkable contortions of the dark marking were seen as the line-shifter was rocked between the extremities of the scale on each side of the zero, which had been brought back to about

* For a description, see *A.J.*, **70**, 284, 1929.

the middle of the scale. A heavy black marking was visible at the setting of the line-shifter equivalent to − 145 km./sec., but it was clear that motions of recession had also developed.

These contortions, or sinuosities, which represented a succession of dark markings or parts of a very complex marking moving with differing radial velocities, made up a rough figure of the letter **S**. Measures of radial velocity made near the north and at the south ends of the formation gave the following values in km./sec., rounded off to the nearest 5 km./sec.:—

Time, G.C.T.	North End.	South End.
h m		
10 59	+110	−160
10 53	+ 80	−110
10 56	+ 85	− 75
10 58	+ 60	Cloud.

At the time of the last observation it was seen that the brilliant patches of flocculi were of about the same intensity, but had separated appreciably. Although no measures of radial velocity were taken of these, it was noted that on turning the line-shifter they appeared most bright when the general field was about at its darkest, *i.e.* corresponding to the centre of the Hα line.

The region of the eruption was observed again at 13^h on November 26, but there was nothing unusual. The brilliant flocculi that had appeared during the eruption on November 25 had faded, but were probably represented by an area of ordinary intensity following that surrounding the sunspot.

The above observations were made alternately by Mr. Greaves and Mr. Newton. About $10^h\ 49^m$ Dr. Jackson made a general examination of the phenomena.

4. It may be added that the magnetic traces were considerably disturbed on November 24 and 25, the total ranges at Abinger being 32′ in D, 185γ in H, and 110γ in V. Magnetic disturbances of the magnitude of small storms occurred on September 3–4, September 28–30, and October 25–30, *i.e.* at approximate intervals of a solar synodic rotation. But in view of the fact that the disturbance of November 24–25 began before the solar phenomena described above, it seems clear that there is no direct causal connection between the two occurrences.

5. This observation is an example of the great utility of the spectrohelioscope in detecting and following hydrogen markings with large and variable radial velocities.

Royal Observatory, Greenwich:
1930 *November* 28.

On the Relation between Colour and Spectral Type in the Different Galactic Latitudes. By E. A. Kreiken.

When the mean relation between colour and spectral type is derived for a selected number of stars, the relation thus obtained will, to a certain extent, depend on the way in which the stars are selected. Now it is evident that, when in different galactic latitudes stars up to one and the same limiting apparent magnitude are considered, the actual mixture of those stars is not identical in the different latitudes.

From a paper by B. Fessenkoff * it appears that differences systematically depending on latitude do occur. By considering the colour equivalents derived from Parkhurst's Yerkes Aktinometrie † Fessenkoff draws the following conclusions: (*a*) with the earlier spectral types the stars in the galactic regions are of a lower temperature (positive colour excess) than those in the higher latitudes; (*b*) with the later spectral types the reverse is the case (negative colour excess). Fessenkoff also considers the material collected by E. Hertzsprung.‡ Here, again, a positive colour excess is obtained for the early types, but with the later ones also the colour excess is generally a positive one. From the material used in the present discussion we also conclude that with the stars of low galactic latitude a positive colour excess occurs. The numerical amount of this excess is largest with the early type stars and decreases with the later ones.

Of the effects which may cause an increase of the mean colour with decreasing (galactic) latitude we consider the following:—

(1) The temperatures of the stars in the lower galactic regions are smaller than in the higher latitudes (see Fessenkoff).

(2) In the lower galactic regions the colours of the stars are affected by a selective absorption in space (see Trumpler §).

(3) The mean absolute magnitude of the stars apparently brighter than a given limit decreases with decreasing latitude.

It is a well-known fact that within each spectral type the colour of the stars systematically depends on absolute magnitude. So there can be no doubt that a part of the colour excesses at least is due to the effect of absolute magnitude. The material used in the present paper was borrowed from the catalogue compiled by Öpik.‖ In his Tables I., II., and III. Öpik gives the means of the colour indices derived by different authors. The different colour systems were reduced to one and the same standard system by non-linear relations.¶ In his Table IV. Öpik also gives colour indices based on Schwarzschild's Göttinger Aktinometrie.** Öpik has tried to reduce the colour-indices of the G.A. for the stars in different parts of the sky to one and the same

* *A.N.*, No. 5686, 375, 1929.
† *Ap. J.*, **36**, 169, 1912.
‡ *Annalen Sterrenwacht Leiden*, Deel, **14**, 1, 1922.
§ *L.O.B.*, No. 420, 1930.
‖ *Publ. Astr. Obs. Tartu*, tome xxvii., No. 1, 1929.
¶ *Cf.* A. Brill, *A.N.*, **223**, 105, 1924.
** *Abh. der Köng. Ges. Wiss. Göttingen*, Neue Folge, **8**, 4, 1912.

system. He therefore applies corrections which systematically depend on the R.A. As the stars in the Göttinger Aktinometrie are contained within relatively narrow limits of declination, this implies that the corrections also depend on galactic latitude, and so it is to be feared that a part of the colour excess will have been obliterated. For the present purpose the values in his Table IV. are therefore of small value.

The stars in his Tables I., II., and III., for which spectroscopic parallaxes are given, were grouped according to spectral type, galactic

TABLE I.

Mean Colour Index for Different Spectral Types and Different Absolute Magnitudes.

Spectrum.	M.	Mean Colour Index and Weight. Latitude. 0–10.	10–20.	20–90.	Combined Mean.
Oe5	$<-4\cdot0$	$-\cdot17$ (2)	$-\cdot42$ (6)	..	$-\cdot35$
	$-4\cdot0-1\cdot0$	$-\cdot23$ (11)	$-\cdot09$ (5)	$-\cdot38$ (4)	$-\cdot23$
Bo	$-3\cdot0--2\cdot0$	$+\cdot03$ (11)	$-\cdot28$ (8)	$-\cdot35$ (14)	$-\cdot20$
	$-2\cdot0--1\cdot0$	$+\cdot01$ (2)	..	..	..
B1	$-3\cdot0--1\cdot0$	$-\cdot22$ (4)	$-\cdot19$ (19)	$-\cdot37$ (4)	$-\cdot22$
B2	$-3\cdot0--1\cdot0$	$-\cdot06$ (17)	$-\cdot26$ (9)	$-\cdot36$ (10)	$-\cdot19$
B3	$-3\cdot0--2\cdot0$	$-\cdot36$ (4)	$-\cdot42$ (9)	$-\cdot35$ (13)	$-\cdot37$
	$-2\cdot0--1\cdot0$	$-\cdot28$ (32)	$-\cdot34$ (8)	$-\cdot34$ (8)	$-\cdot31$
	$-1\cdot0-0\cdot0$	$-\cdot30$ (14)	$-\cdot19$ (8)	$-\cdot23$ (11)	$-\cdot25$
B5	$-2\cdot0--1\cdot0$	$-\cdot24$ (28)	..	$-\cdot25$ (5)	$-\cdot24$
	$-1\cdot0-0\cdot0$	$-\cdot26$ (14)	$-\cdot23$ (12)	$-\cdot26$ (18)	$-\cdot25$
	$0\cdot0-+1\cdot0$	..	$-\cdot25$ (4)	$-\cdot20$ (8)	$-\cdot22$
B8	$-1\cdot0-0\cdot0$	..	$-\cdot23$ (5)	..	$-\cdot23$
	$0\cdot0-+1\cdot0$	$-\cdot25$ (12)	$-\cdot25$ (4)	$-\cdot27$ (17)	$-\cdot26$
B9	$-1\cdot0-+1\cdot0$	$+\cdot08$ (7)	..	$-\cdot22$ (10)	$-\cdot09$
A0	$-1\cdot0-0\cdot0$	..	$-\cdot22$ (6)	$-\cdot21$ (2)	$-\cdot22$
	$0\cdot0-+1\cdot0$	$-\cdot12$ (21)	$-\cdot11$ (9)	$-\cdot17$ (65)	$-\cdot15$
A2	$0\cdot0-+1\cdot0$	$-\cdot10$ (8)	$-\cdot12$ (2)	$-\cdot10$ (37)	$-\cdot10$
	$+1\cdot0-+2\cdot0$	$-\cdot04$ (10)	$-\cdot02$ (17)	$-\cdot08$ (51)	$-\cdot05$
A5	$0\cdot0-+1\cdot0$	..	$\cdot00$ (6)	..	$\cdot00$
	$+1\cdot0-+2\cdot0$	$+\cdot03$ (8)	..	$+\cdot07$ (56)	$+\cdot06$
F0	$<-1\cdot0$	..	..	$+\cdot21$ (1)	$+\cdot21$
	$-1\cdot0-0\cdot0$	..	..	$+\cdot22$ (8)	$+\cdot22$
	$0\cdot0-+1\cdot0$	$+\cdot26$ (4)	..	$+\cdot20$ (25)	$+\cdot21$
	$+1\cdot0-+2\cdot0$	$+\cdot29$ (8)	$+\cdot21$ (10)	$+\cdot12$ (46)	$+\cdot15$
	$+2\cdot0-+3\cdot0$	$+\cdot21$ (42)	$+\cdot23$ (6)	$+\cdot21$ (27)	$+\cdot21$
F5	$<-1\cdot0$	$+\cdot31$ (7)	..	..	$+\cdot31$
	$-1\cdot0-0\cdot0$	$+\cdot28$ (15)	..	$+\cdot73$ (5) ?	$+\cdot40$

TABLE I.—*continued.*

Spectrum.	M.	Mean Colour Index and Weight. Latitude. 0–10.	10–20.	20–90.	Combined Mean.
F5	0·0 – +1·0	+ ·43 (10)	..	..	+ ·43
	+1·0 – +2·0	+ ·30 (16)	..	+ ·31 (16)	+ ·31
	+2·0 – +3·0	+ ·24 (14)	+ ·31 (17)	+ ·44 (2)	+ ·28
	> +3·0	+ ·28 (7)	+ ·32 (82)	+ ·25 (15)	+ ·31
F8	< −1·0	+ ·88 (6)	..	+ ·52 (7)	+ ·73
	−1·0 – +1·0	..	..	..	..
	+1·0 – +2·0	..	..	+ ·34 (2)	+ ·34
	+2·0 – +3·0	..	..	+ ·46 (18)	+ ·46
	+3·0 – +4·0	+ ·44 (17)	+ ·39 (7)	+ ·37 (72)	+ ·39
G0	< −1·0	+ ·78 (7)	..	+ ·85 (12)	+ ·82
	−1·0 – 0·0	+ ·66 (3)	+1·02 (2)	+ ·81 (30)	+ ·81
	0·0 – +1·0	+ ·75 (10)	..	..	+ ·81
	+1·0 – +2·0	+ ·70 (2)	..	+ ·80 (7)	+ ·78
	+2·0 – +3·0	+ ·56 (24)	..	+ ·76 (7)	+ ·60
	+3·0 – +4·0	+ ·49 (72)	+ ·44 (13)	+ ·49 (15)	+ ·49
G5	< −1·0	+1·52 (8)	..	+ ·86 (12)	+1·12
	−1·0 – 0·0	..	+ ·86 (14)	+ ·92 (39)	+ ·90
	0·0 – +1·0	..	+ ·72 (12)	+ ·86 (14)	+ ·80
	+1·0 – +2·0	+ ·78 (9)	..	+ ·84 (17)	+ ·81
	+2·0 – +3·0	+ ·76 (2)	..	+ ·59 (5)	+ ·69
	> +3·0	..	− ·56 (5)	− ·59 (35)	− ·59
K0	< −1·0	+1·71 (14)	..	+1·52 (12)	+1·60
	−1·0 – 0·0	+1·22 (15)	+1·31 (28)	+1·16 (54)	+1·21
	0·0 – +1·0	+1·02 (27)	+1·10 (38)	+1·07 (223)	+1·07
	+1·0 – +2·0	+1·01 (53)	+1·11 (43)	+1·11 (110)	+1·08
	+2·0 – +3·0	+ ·99 (8)	+1·07 (25)	+1·13 (39)	+1·09
	> +3·0	+ ·90 (8)	+ ·85 (29)	+ ·79 (35)	+ ·83
K2	< −1·0	+2·14 (5)	+2·01 (1)	+1·59 (10)	+1·79
	−1·0 – 0·0	+1·68 (16)	+1·63 (8)	+1·43 (8)	+1·60
	0·0 – +1·0	+1·41 (2)	+1·49 (2)	+1·44 (7)	+1·44
	+1·0 – +2·0	..	..	+1·30 (18)	+1·30
	+1·0 – +3·0	..	..	..	..
	> +3·0	+1·04 (5)	..	+1·52 (4)	+1·25
K5	< −1·0	+1·90 (7)	..	..	+1·90
	−1·0 – 0·0	+1·49 (4)	+1·69 (7)	+1·65 (16)	+1·64
	0·0 – +1·0	+1·64 (8)	+1·77 (19)	+1·66 (88)	+1·68
	+1·0 – +2·0	..	+1·58 (2)	..	+1·58
Ma	< −2·0	+2·04 (25)	+2·15 (13)	+1·66 (12)	+1·98

TABLE I.—*continued.*

Spectrum.	M.	Mean Colour Index and Weight. Latitude. 0–10.	10–20.	20–90.	Combined Mean.
Ma	−2·0 – −1·0	+1·66 (8)	..	+1·80 (6)	+1·72
	−1·0 – 0·0	+1·70 (12)	+1·72 (14)	+1·73 (58)	+1·72
	0·0 – +1·0	..	..	+1·70 (26)	+1·70
	+1·0 – +2·0	..	..	+1·62 (5)	+1·62
	> +3·0	(3 stars from Gött. Akt.)			+1·55
Mb	< −2·0	..	+1·53 (7)	+1·36 (13)	+1·42
	−2·0 – −1·0	+1·52 (5)	+1·33 (11)	+1·55 (2)	+1·40
	−1·0 – 0·0	..	..	+1·24 (2)	+1·24
	0 – +2·0	+1·37 (1)	..	+1·60 (5)	+1·56
	> +3·0	(2 stars from Gött. Akt.)			+1·45

latitude, and absolute magnitude. Within each group the mean colour-index was determined. The results appear in Table I., where also (in brackets) the weights are given. According to Öpik the probable error corresponding to unit weight is $\pm 0^m{\cdot}06$. Owing to the rapid change of colour within each spectral type the various sub-classes were treated separately. For the different galactic belts the following limits were adopted: 0° to 10°, 10° to 20°, and 20° to 90° of northern and southern declination. The limits were chosen in this way with a view to the consideration of the results recently published by R. Trumpler.* In his paper Trumpler concludes that objects lying within 8° of the galactic equator are affected by a total as well as by a selective absorption. Designating by K_v the absorption constant per 1000 parsecs for visual rays and by K_p that for the photographic observations, the change of colour is

$$C = K_p - K_v = +0{\cdot}32.$$

If Trumpler's results are correct this selective absorption may also influence the colour-indices of the early type stars used here.

The influence of galactic latitude on the mean relation of colour and spectral type may be studied in the following way. In the different latitudes and for each spectral sub-class separately we derive from the numbers in Table I. the weighted mean colour-index for stars of all absolute magnitudes together. With the earlier spectral types we also take into account some additional stars for which no spectroscopic parallaxes are available. With the later types a crude subdivision into giants and dwarfs is made. With the types Fo and F2, all stars with $M > 2{\cdot}0$ were considered as dwarfs; with all later types, $M = 3{\cdot}0$ was adopted as the dividing absolute magnitude. For the latitudes 0° to 10° and 20° to 90° the results are graphically represented in fig. 1. From the figure it appears that for the highly luminous stars a colour excess occurs in the lower latitudes. With the dwarf stars the colour

* *L.O.B.*, No. 420, 1930.

apparently is a constant. From this latter fact we conclude that no appreciable error systematically depending on latitude affects the

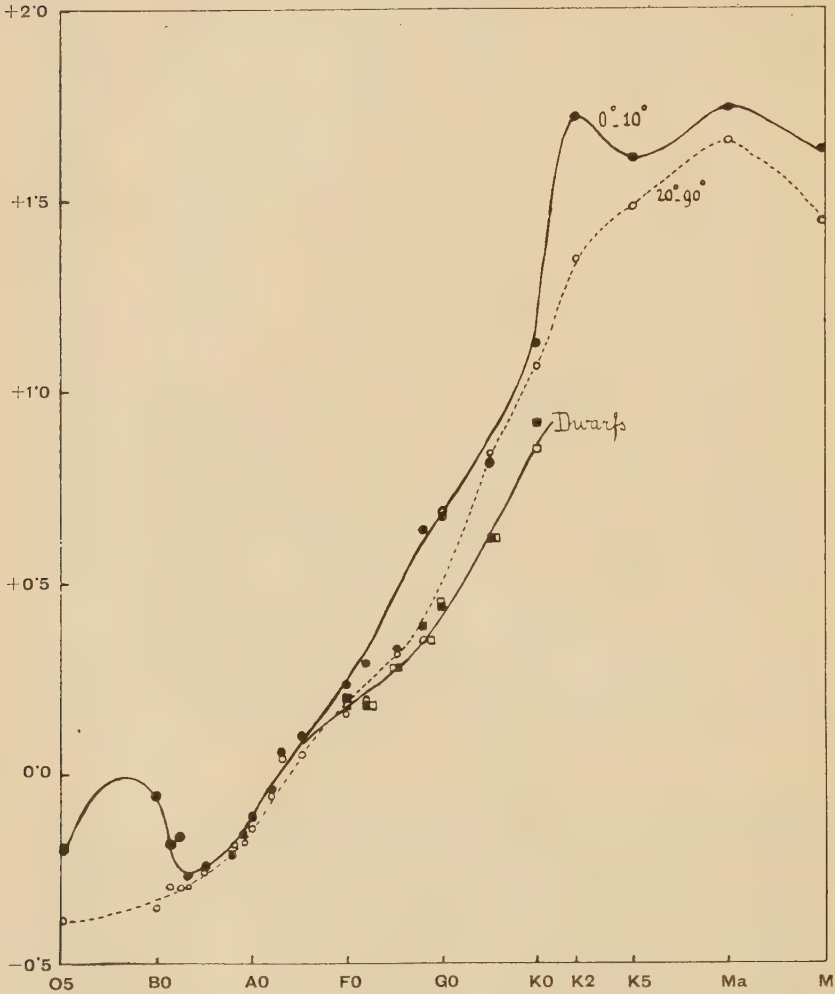


Fig. 1.—Relation between mean colour-index and spectral type in different galactic latitudes.

different sets of colour equivalents. Consequently the colour excess of the brighter stars in the galactic regions must be considered as real, *i.e.* it must be due to one of the effects mentioned under (1), (2), and (3). We now consider the effect of absolute magnitude. In order to have sufficient material the results for all galactic latitudes were combined into one set.

I do not think there is any serious objection to doing this, as the direct effect of absolute magnitude cannot systematically depend on

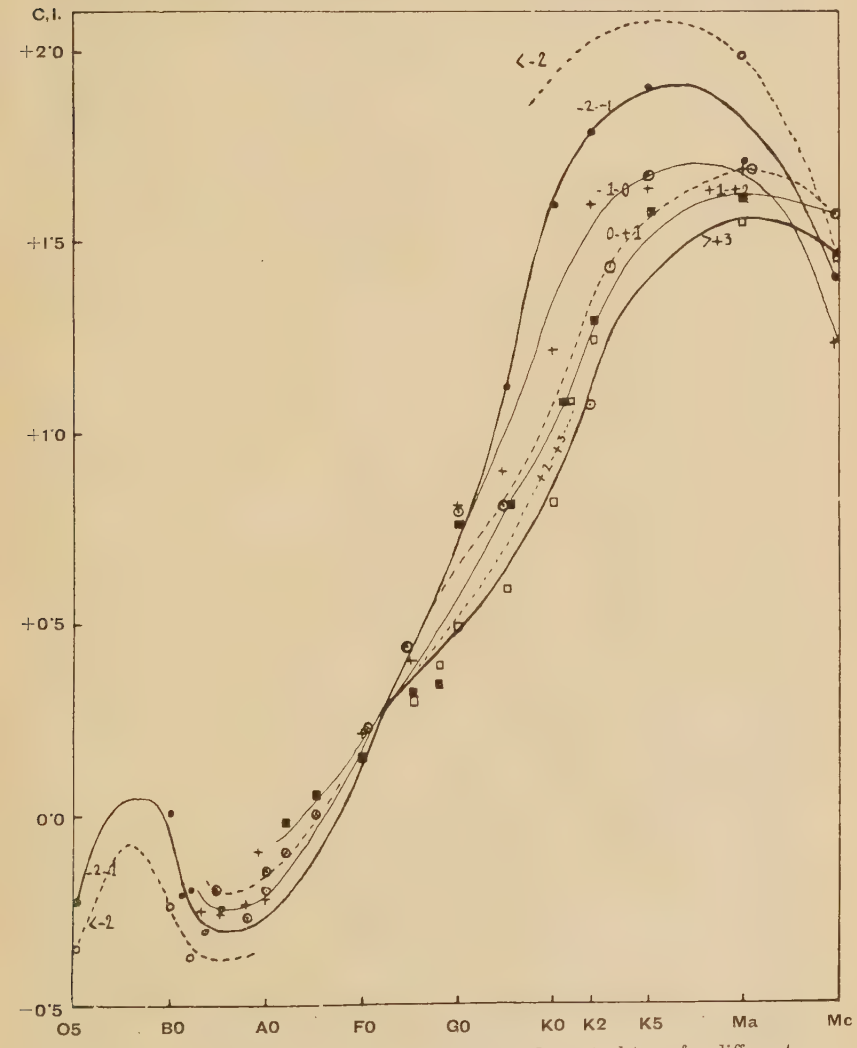


Fig. 2.—Relation between mean colour-index and spectral type for different absolute magnitudes.

latitude. For the different spectral types and for the different absolute magnitudes the weighted mean colour-indices also appear in Table I. (combined mean). From these latter values fig. 2 was drawn. In this figure the mean colour-indices corresponding to the different absolute magnitudes are plotted against spectral type, and through the points

obtained in this way smooth curves are drawn. Fig. 2 graphically represents the results previously obtained by various authors.* It appears that with the early spectral types a low colour-index (negative colour excess) corresponds to a high absolute luminosity. With the later B stars the range of the colour-index decreases. With the types A3 and Fo the curves corresponding to the different absolute magnitudes apparently intersect. From our material we cannot decide whether all curves intersect in one and the same point or in different points. Anyhow it is evident that here the change of colour-index with absolute magnitude is very small. With the types later than Fo the change of colour systematically depending on absolute magnitude is again very pronounced, but now a positive colour excess corresponds to a high absolute luminosity. The range of colour-index continuously increases with the later spectral types. The maximum range occurs with the K stars. With the M stars the range in colour decreases and apparently the different curves intersect again with the M*b* stars. With the M*b* stars, however, the distribution of the individual values is rather irregular and the weights of the colour-indices are low. The general increase of colour-index with spectral type is well indicated in our figure. At the extreme ends of the curve, however, irregularities do occur. As to the fluctuations which occur with the M stars, these have previously been noted by other authors and probably are caused by absorption bands. With the O stars and the early B stars the weights are low, and it is difficult to decide whether the large fluctuations in the mean curve are real or due to the disturbing influence of extremely coloured supergiants. The phenomenon is rather persistent and also appears in fig. 1. (It should be remarked that these same irregularities occur if, instead of the colour-indices given by Öpik, the colour equivalents derived by Hertzsprung † are used.) The well-known difference between the colours of the giants and dwarfs of the same spectral type is also very evident. With the M stars this difference apparently is very small or perhaps even zero.

Notwithstanding the remaining uncertainties we may read, with sufficient accuracy, from our smoothed curves in fig. 2, the numerical amount of the change of colour with absolute magnitude within each spectral type. These amounts are given in Table II. The values given there indicate the corrections which should be applied to the different mean colour-indices in order to reduce them to the mean colour-index corresponding to the absolute magnitude − 1 to 0. With the types O–B2 this would imply an extrapolation of the observed curve, and therefore here the corrections necessary for a reduction to the absolute magnitude − 2 to − 1 are given. For evident reasons no values have been entered either for the A3–Fo or the M*b* stars.

* See, for instance, Bottlinger, *Veröff. Babelsberg*, Bd. **3**, H. 4, 1923.

† Nearly all stars used by Hertzsprung are also contained in Öpik's Catalogue. Hertzsprung used linear relations for reducing the different colour systems to one and the same standard system. Non-linear relations were used by Öpik. So, according to Brill (*loc. cit.*), we expect Öpik's values to be the better ones. Therefore Öpik's colour equivalents were used throughout.

TABLE II.

Corrections to be applied to the Mean Colour-indices of the different Spectral Types and different Absolute Magnitudes in order to reduce them to one and the same Magnitude.

Spectrum. \ M_{abs}	<-2.	-2 to -1.	0 to $+1$.	$+1$ to $+2$.	$+3$.
Oe5	+·07	..	..	..	..
B0	+·05	..	..	..	..
B1	+·06	..	..	..	..
B2	+·08	..	..	..	..
B3	+·12	+·06	..	..	..
B5	+·08	+·05	−·02	..	..
B8	+·05	+·02	−·02	..	..
B9	..	+·02	−·02	..	..
A0	..	+·01	−·02	..	..
A2	..	·00	−·01	..	..
F5	..	·00	+·01	+·02	−·02
F8	..	−·04	+·07	+·13	−·06
G0	..	−·06	+·06	+·12	−·07
G5	..	−·22	+·05	+·10	−·02
K0	..	−·32	+·18	+·26	−·09
K2	−·28	−·20	+·14	+·26	−·14
K5	−·28	−·19	+·05	+·10	−·11
Ma	−·26	−·09	+·04	+·09	−·02

(For the O5–B2 stars the correction to M_{abs} -2 to -1 is given. For all stars fainter than $+2{\cdot}0$, to M_{abs} $+2{\cdot}0$ to $+3{\cdot}0$. For all other stars, to M_{abs} $-1{\cdot}0$ to $0{\cdot}0$.)

We now consider the question, whether the effect of absolute magnitude completely explains the colour excesses in the low galactic regions. Therefore the corrections given in Table II. were applied to the individual values of Table I. Then for the different latitudes final mean values of the colour-index are obtained by combining within each spectral type the reduced colour-indices of the stars with $M < +2{\cdot}0$ into one set. A second mean value was obtained from the stars with $M > +2{\cdot}0$. From these final mean values we easily derive the remaining colour excesses of the stars in the galactic zones (0° to 10° and 10° to 20°) over those in the zone 20° to 90°. Evidently they are equal to the differences between the reduced colour-indices. Owing to their relatively small weights the run of these differences is rather irregular. Therefore all differences obtained for the stars earlier than B5 were combined into one set and the weighted mean difference was computed. The same was done with the B8–A5, the F0–F8, the G0–G5, and the K0–K5 stars. With the types later than F0 the stars with $M < +2{\cdot}0$ and those with $M > +2{\cdot}0$ were separately treated. We thus obtain the numbers on the left side in Table III. From this table it appears that there remains a small but very persistent colour excess. This remaining excess is largest with the B stars and gradually

decreases with the later types. Among the Fo–K5 stars with $M < -1$ and the M*a* stars with $M > -2$ there are, however, several supergiants, so it is possible that with the later types part of the effect at least is due to extremely coloured supergiants. Consequently a second solution was made in which the groups of stars just mentioned were excluded. The results of this second solution are given on the right side in Table III. The results for the stars with $M > +2{\cdot}0$, of course, remain unaltered. We again obtain a run in the numerical values of the colour excess systematically depending on spectral type, but now the colour excess for the later types is negative. Comparing the values on the right and left sides in Table III. it appears that the exclusion of a few stars has an enormous influence on the numerical values of the remaining colour excesses, so it is at least doubtful if the differences are to be ascribed to a selective space absorption, although the general run of the numbers is such as we would expect from Trumpler's results.

Table III.

Remaining Weighted Mean Colour Excesses of the Stars in the Galactic Regions I. and II. (0° *to* 10° *and* 10° *to* 20°) *over those in the Region III.* (20° *to* 90°). *In Brackets the Weights are given.*

$M_{abs.}$ Spectrum.	M<+2·0.		M>+2·0.		M<+2·0. Supergiants excluded.	
	I.–III.	II.–III.	I.–III.	II.–III.	I.–III.	II.–III.
Bo–B5	+·10 (57)	+·03 (40)	..	..	+·10 (57)	+·03 (40)
B8–A5	+·04 (50)	+·03 (39)	..	..	+·04 (50)	+·03 (39)
Fo–F8	+·04 (29)	+·06 (9)	+·02 (40)	+·03 (28)	+·01 (24)	+·06 (9)
Go–G5	+·05 (29)	−·01 (22)	−·04 (20)	·16 (12)	−·05 (19)	·07 (21)
Ko–K5	+·02 (107)	+·07 (117)	−·08 (15)	−·01 (31)	−·02 (95)	+·07 (113)
M*a*	+·03 (32)	+·11 (22)	..	..	−·11 (17)	−·01 (12)

The stars here considered are on the whole very near, so that the effect of absorption must be small. With a view to the apparent magnitude equation and the seasonal effects * which may influence the colour-indices it would, however, be dangerous to base any definite conclusions on the numbers of Table III.

Summarising, we have :

1. The mean relation of colour and spectral type systematically depends on galactic latitude.

2. The colour excesses in the galactic regions are almost entirely due to the effect of absolute magnitude.

3. After applying corrections for absolute magnitude there remains a small colour excess in the low latitudes, which systematically varies with spectral type. These remaining excesses are very small, and no general conclusions as to selective absorption in space can be drawn from them.

* See, for instance, Miss Williams, *Harvard Circular*, p. 348, 1930.

Lembang, Java :
July 1930.

Note on the Axial Rotation of the Stars. By E. A. Kreiken.

In a recent paper C. T. Elvey * has determined for 59 stars the velocity of axial rotation, or rather its component in the line of sight. From this velocity the angular velocity and the period of axial rotation of the outer layers of each star might be determined if the diameters of the stars were known. For the majority of the stars in Elvey's list the hypothetical semi-diameters have been determined by E. Hertzsprung.† They are given in Table I. The first column of this table gives the name of the star, the second contains the hypothetical semi-diameter as computed by Hertzsprung (unit 0″·0001), the third the velocity of rotation observed by Elvey, and the fourth gives the spectral type. The parallaxes are given in the fifth column. Generally, spectroscopic parallaxes were used. If these were not available, trigonometric (tr.), proper motion (p.m.) parallaxes, or parallaxes derived from absolute magnitude (M) were used. These cases are indicated in the table.

From Hertzsprung's values and from the parallaxes we find the values the semi-diameters would have if the stars were placed at a distance corresponding to a parallax of 0″·1. The reduced hypothetical semi-diameters are given in column six of the table. In the next column the reduced velocities of rotation are given, *i.e.* the velocities corresponding to a reduced semi-diameter 10. (The only reason for adopting this value 10 is that it is convenient for the computations.) These latter quantities are proportional to the angular velocities of rotation and inversely proportional to the periods of rotation.

Table I.

List of Stars used in the Present Investigation.

Star.	Hyp. Semi-diam. (Unit 0″·0001).	Velocity of Rotation. (km./sec.).	Spectral Type.	π.	D (reduced).	Reduced Velocity of Rotation. km./sec.	Period in Days if Binary.
(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).
λ Ori.	3	200	O8	″·006	50	40	
10 Lac.	1	70	O9	·003 (M)	33	21	
φ' Ori.	2	0	Bo	·006	33	0	
ϱ Leo.	2	24	Bo*p*	·006	33	7	
ξ Per.	7	50	B1	·007	100	5	
γ Peg.	4	0	B1	·007	60	0	
γ Ori.	7	70	B2	·018	39	18	
ι Her.	3	0	B3	·005	60	0	9·5
π^4 Ori.	3	25	B3	·006	50	5	

* *Ap. J.*, **71**, 4, 221, 1930.
† *Annalen Sterrenwacht Leiden*, Deel xiv., eerste stuk, 1922.

TABLE I.—*continued.*

Star.	Hyp. Semi-diam. (Unit 0″·0001).	Velocity of Rotation (km./sec.).	Spectral Type.	π.	D (reduced).	Reduced Velocity of Rotation. km./sec.	Period in Days if Binary.
(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).
ε Cas.	4	25	B3	″·017	24	10	
η Lyr.	2	0	B3	·007	28	0	
ζ Cas.	3	25	B3	·005	60	4	
102 Her.	2	70	B3	·008 (M)	25	28	
ε Del.	3	70	B5	·010	30	23	
ε Tau.	2	100	B5	·013	15	66	
τ Her.	3	25	B5	·013	23	11	
ξ Dra.	4	0	B5	·014	29	0	
67 Oph.	3	25	B5*p*	·007	43	6	
β Per.	9	70	B8	·045	20	35	2·9
λ Aql.	4	200	B9	·038 (tr.)	11	182	
α Lyr.	19	0	A0	·123	15	0	
α CBr.	6	125	A0	·052	12	100	17·4
β UMa.	7	0	A0	·054	13	0	
θ Aql.	4	0	A0	·019	21	0	17·1
α^1 Gem.	11	25	A0	·066	17	15	2·9
γ Gem.	9	0	A0	·063	14	0	
β Aur.	8	0	A0*p*	·055	15	0	4·0
ε UMa.	11	25	A0*p*	·069	16	16	
α^2 CVn.	7	25	A0*p*	·035	20	12	5·5
η Leo.	3	0	A0*p*	·009	33	0	
β Leo.	9	100	A2	·079	11	90	
ξ^2 UMa.	8	25	A2	·054	15	17	
δ UMa.	4	200	A2	·042 (tr.)	10	200	
γ UMi.	5	150	A2	·047	10	150	0·1
α Cyg.	13	0	A2*p*	·010 (p.m.)	30	0	
δ Leo.	8	130	A3	·066	12	110	
111 Her.	3	70	A3	·034	9	78	
ι UMa.	8	100	A5	·066	12	80	
β Tri.	7	70	A5	·050	14	50	31·4
α Aql.	20	250	A5	·145	14	180	
γ Vir.	10	25	F0	·076	13	19	
δ Aql.	8	100	F0	·058	14	70	
δ Gem.	8	70	F0	·055	15	67	
α CMi.	31	0	F5	·331	9	0	
α Per.	20	0	F5	·048	42	0	
41 Cyg.	6	0	F5	·010	60	0	
ε Aur.	13	0	F5	·017	76	0	
ι Pis.	6	0	F8	·100	6	0	

One glance at the figures of Table I. proves that the reduced velocities of axial rotation (the angular velocities) systematically depend on the values of the reduced semi-diameters. This is still more evident from the numbers in Table II. In this table the stars are grouped according to the values of their reduced semi-diameters. Within each group the mean values D, of the reduced semi-diameters, and R, of the reduced velocities of axial rotation, are given. It appears that the angular velocities rapidly increase with decreasing D.

TABLE II.

The Amount of Rotation R as a Function of the Reduced Hypothetical Semi-diameter D.

D.	Mean D.	Mean R. km./sec.	Number of Stars.	log $R_{obs.}$	log $R_{calc.}$	O–C.	Period in Days.
80+	115	2·5	2	0·398	0·29	+·11	49
40–80	56	6·1	9	0·785	0·75	+·03	20
30–39	33	11·5	6	1·06	1·09	−·03	11
20–29	24	12	8	1·08	1·30	−·22	10
15–19	15	26	7	1·415	1·59	−·17	4·7
11–14	13	80	11	1·90	1·70	+·20	1·5
10–	9	86	5	1·935	1·93	·00	1·4

The relation between the logarithms of the mean angular velocities and the mean hypothetical (reduced) semi-diameters is almost a linear one. If we put

$$\log R = a + b \log D$$

we find from a least-squares solution $a = +3{\cdot}34$ and $b = -1{\cdot}49$, and so

$$\log R = 3{\cdot}34 - 1{\cdot}49 \log D \quad . \quad . \quad . \quad (1)$$

In Table II. the observed and computed values of log R are compared. For a few stars * of Hertzsprung's list the actual diameters are known from interferometer measurements. These stars are entered in Table III. In this same table the observed diameters (adopting for the astronomical unit of distance the value 15×10^7 km.), the hypothetical semi-diameters given by Hertzsprung, and the reduced values D are given.

TABLE III.

Stars with known Diameters.

Star.	Diameter (Unit = 15×10^7 km.).	Hypothetical Semi-diameter.	π.	Reduced Semi-diameter.
			″	
o Cet.	3·30	..	·017	
α Ori.	2·67	211	·017	1024
α Tau.	0·37	176	·057	309
α Boo.	0·30	123	·080	154

* Michelson and Pease, *Publ. A.S.P.*, **89**, 1925.

From the table we find $1487D = 50{\cdot}1 \times 10^7$ km. or

$$10D = 33{\cdot}6 \times 10^5 \text{ km.}$$

Consequently the actual diameter of a star with reduced hypothetical diameter 10 is equal to $33{\cdot}6 \times 10^5$ km., and its circumference will be $105{\cdot}5 \times 10^5$ km. So for a reduced velocity of axial rotation R = 1 km./sec., the period of rotation will be

$$\frac{105{\cdot}5 \times 10^5}{24 \times 36 \times 10^2} = 122 \text{ days} \quad . \quad . \quad . \quad (2)$$

From the value (2) we derive the periods of rotation for the different groups of stars in Table II. (As they are derived from the component in the line of sight the actual periods will be somewhat shorter still.)

We put $R = k/P$ where P indicates the period of rotation; then from (2) we find $\log k = 2{\cdot}09$. So from formula (1) we find that the relation between the period of rotation and the reduced semi-diameter is given by

$$\log P = 1{\cdot}49 \log D - 1{\cdot}25 \quad . \quad . \quad . \quad (3)$$

It is of some interest to compare the periods of axial rotation as derived from (3) with the observed periods of orbital revolution of the spectroscopic binaries. According to Jeans * and G. Shajn and O. Struve † we must expect that immediately after fission (if a double star originates by fission) the periods of revolution and of axial rotation will be identical. For the present we consider only the spectroscopic binaries in Elvey's list and two eclipsing variables (ϵ Aur. and λ Tau.) for which Hertzsprung gives the hypothetical semi-diameters. A survey of the results appears in Table IV., where for different intervals of the reduced semi-diameter D the means of the observed values $\log P_o$ (period of orbital revolution) are compared with the values $\log P_c$ (period of axial rotation) obtained from (3). The two quantities are roughly proportional. This also appears from the figure, where the logarithms of the periods are plotted against those of the reduced diameters. Black dots represent the logarithms of the mean periods derived from Elvey's values, open circles the mean logarithms of the observed periods of orbital revolution, while formula (3) is represented by a full-drawn line.

TABLE IV.

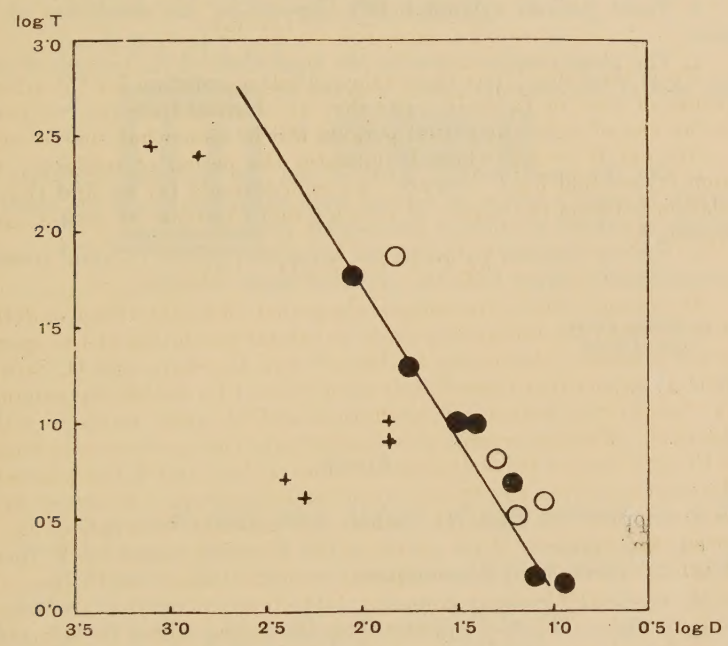
Mean Values of the Differences between the Logarithms of the Periods of Orbital Revolution and of Axial Rotation for Spectroscopic Binaries.

D.	O—C.	Number of Stars.
40–80	+·37	3
20–29	+·11	3
15–19	−·01	2
10–14	+·23	3

* *Astronomy and Cosmogony*, p. 280, 1928.
† *M.N.*, **89**, 222, 1929.

For the spectroscopic binaries the spread of the individual values of log P round the curve is very large. In a paper which will be shortly published these points will be discussed in detail, and it will be shown that large deviations of the observed values of log P only occur for the binaries with a large value of the eccentricity *e*. As all details will be given in the paper just referred to, these details are omitted from the present note.

Finally, the crosses in the figure indicate the logarithms of the



Relation between the logarithms of the mean periods of axial rotation (T) and of diameters (D). Black dots indicate values derived from Elvey's list. Open circles indicate the logarithms of mean observed periods of revolution of spectroscopic binaries. Crosses indicate the logarithms of the observed periods of Cepheid and long-period variables.

observed periods of light variation of some Cepheid and long-period variables which are contained in Hertzsprung's catalogue. The stars in question are η Aql., δ Cep., ξ Gem., α UMi., η Gem. From the values of the semi-diameters given by Hertzsprung we easily find the reduced diameters by means of the parallaxes. For *o* Ceti the reduced diameter is readily found from its actual diameter. For the same reason as before no details are given. Apparently the period of light variation is considerably shorter than the period of axial rotation derived from (3). With the four Cepheid variables the mean logarithmic difference is $-$ 1·07. With the two long-period variables this difference is $-$ 0·73, but here the curve (3) is largely extrapolated. The result

for the Cepheid variables indicates that the period of light variation is either about twelve times shorter than the period of rotation or that the axial rotation of these stars is abnormal.

Summary.

1. From Elvey's list of rotational velocities the periods of the axial rotation of the different stars are computed.

2. These periods systematically depend on the diameters of the stars.

3. The mean relation between the logarithms of the periods of rotation and of the semi-diameters is given by the equation

$$\log P = 1{\cdot}49 \log D - 1{\cdot}25 \quad . \quad . \quad . \quad (3)$$

4. For the spectroscopic binaries and the eclipsing variables the periods of axial rotation as derived from the formula and the observed periods of orbital revolution are roughly proportional.

5. For the Cepheid variables the computed period of axial rotation is considerably larger than the period of light-variation.

Lembang, Java :
1930 *August.*

Errata.

On p. 43, equation (99), *for* $[\phi/b_1)]^{\frac{3}{2}}$ *read* $[\phi(b_1)]^{\frac{3}{2}}$.

On p. 47, equation (108′) should be

$$T_e = \frac{2^{\frac{5}{2}}\pi^{\frac{1}{2}}G^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{4}}\beta^{\frac{1}{2}}}{(ac)^{\frac{1}{4}}(5K)^{\frac{1}{2}}}\left[-\frac{a_1^2 f'(a_1)}{\xi_0^2 f'(\xi_0)}\frac{1}{b_1^5\phi'(b_1)}\right]^{\frac{1}{8}}\left(\frac{a_1}{\xi_0}\right)^{\frac{1}{2}} \quad . \quad (108')$$